Irrelevant solvable deformations with boundaries and defects

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Based on the work

Y. Jiang, F. Loebbert, D. Zhong, 2109.13180

Initiated from the previous workshop

1. Motivations & Reviews

Motivation

TTbar and other irrelevant solvable deformation have been intensely studied in recent years

These studies lead to a deeper understanding about **quantum field theory** and **integrable models**

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QFTs and integrable systems can have extended objects such as **boundaries** and **defects**.

Question How does TTbar and other deformations affect such structures ?

Goal Investigate this question in Integrable QFT

Integrable QFT

QFT described by an action

$$S = \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \, \mathcal{L}$$

Quantize it, and compute observables (perturbatively)

Integrable QFT

QFT described by an action

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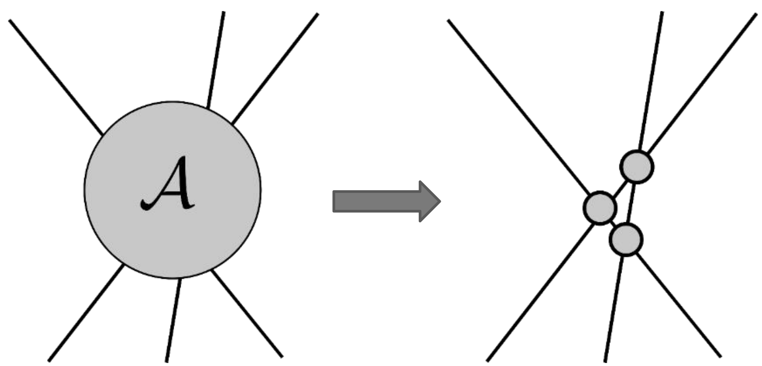
Quantize it, and compute observables (perturbatively)

The bootstrap approach



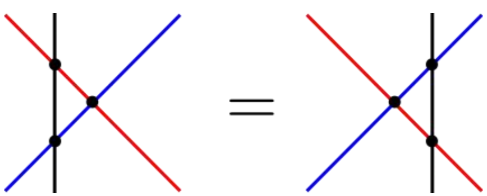
Factorized scattering

- S-matrix encodes dynamical info
- Find S-matrix non-perturbatively
- Compute other observables

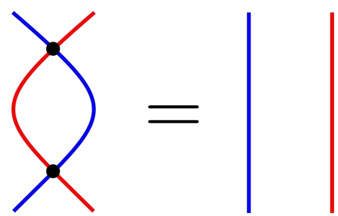
S-matrix bootstrap

The bootstrap axioms

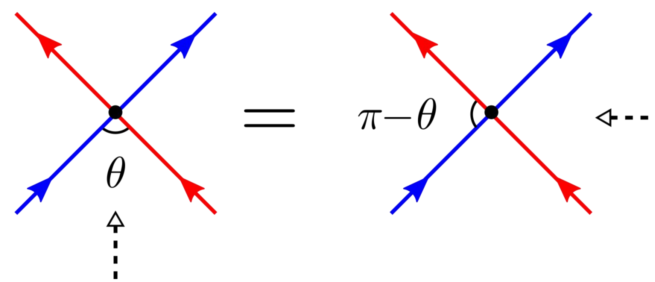
YBE



Unitarity



Crossing



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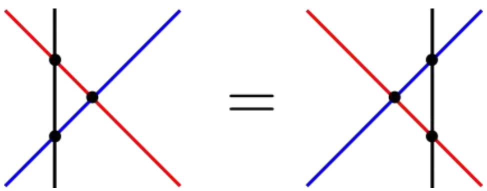

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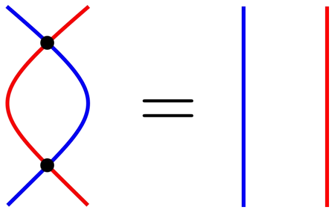

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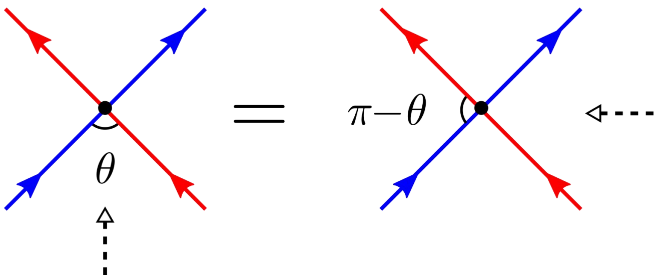

- The importance of **CDD factor** become more clear now
- **Spectrum** can be found by Bethe ansatz and TBA
- **Correlators** can be found by form factor bootstrap

Boundary IQFT

Lagrangian description

$$S_{\mathrm{B}} = \int_{-\infty}^{\infty} \mathrm{d}y \int_{0}^{\infty} \mathrm{d}x \mathcal{L} + \int_{-\infty}^{\infty} \mathrm{d}y \mathcal{L}_{\mathrm{B}}$$

Boundary IQFT

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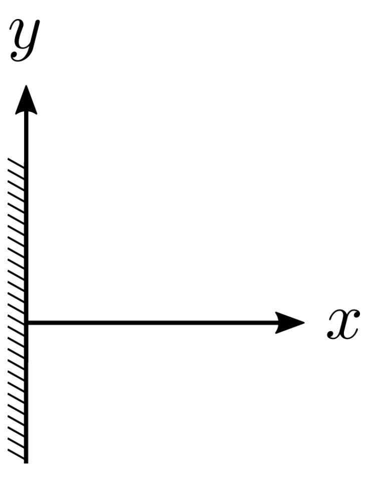

Boundary in space direction

$${}_{\mathrm{B}}\langle 0|\mathcal{O}_1 \cdots \mathcal{O}_n|0\rangle_{\mathrm{B}}$$

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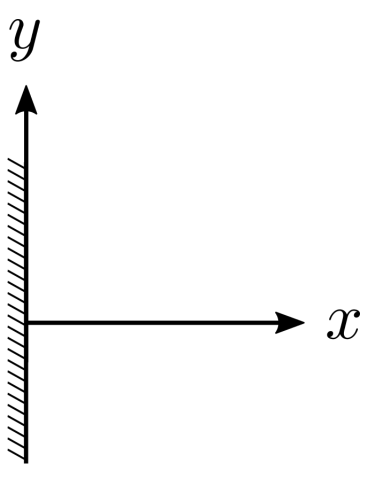

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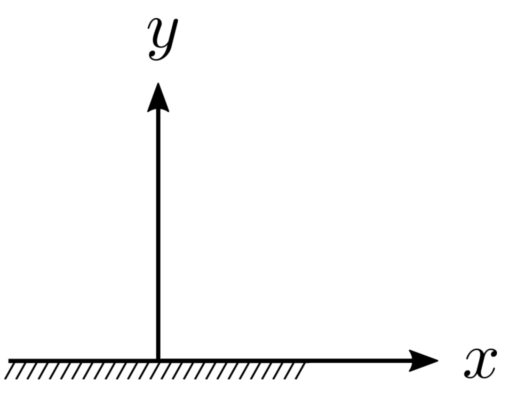

Boundary in time direction

$$\langle 0|\mathcal{O}_1 \cdots \mathcal{O}_n|B\rangle$$

Boundary IQFT

Bulk integrability

[Ghoshal and Zamolodchikov 1993]

There exist infinitely many conserved currents

$$\partial_{\bar{z}} T_{s+1} = \partial_z \Theta_{s-1} \qquad \partial_z \bar{T}_{s+1} = \partial_{\bar{z}} \bar{\Theta}_{s-1}$$

Boundary IQFT

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Integrable boundary

Boundary conditions which preserves integrability

$$\left[T_{s+1} + \bar{\Theta}_{s-1} - \bar{T}_{s+1} - \Theta_{s-1}\right]\Big|_{x=0} = \frac{\mathrm{d}}{\mathrm{d}y}\theta_s(y)$$

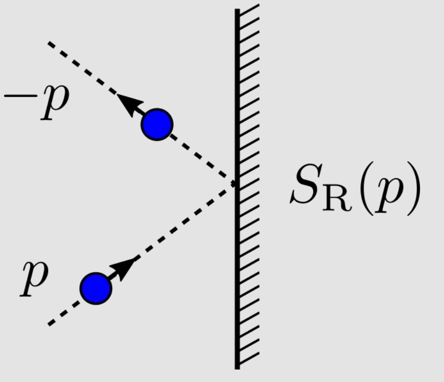
Preserves infinitely many conserved charges

$x = 0$

Boundary IQFT

Bootstrap description

- Scattering is purely elastic
- Even charges conserved
- Described by boundary S-matrix



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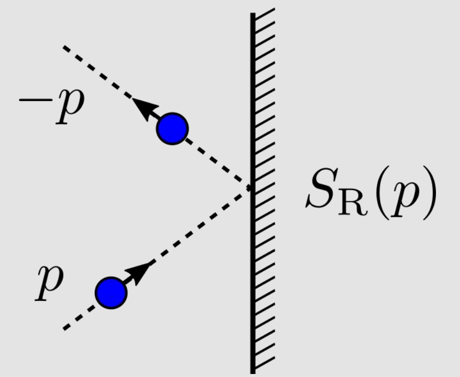

Boundary S-matrices can be determined by **bootstrap axioms**

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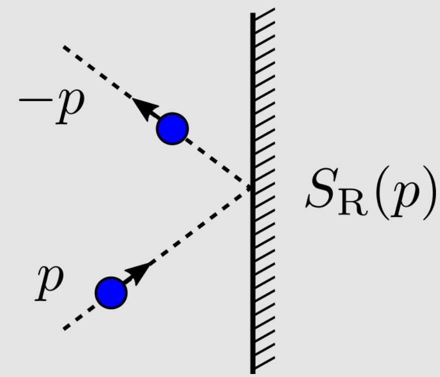

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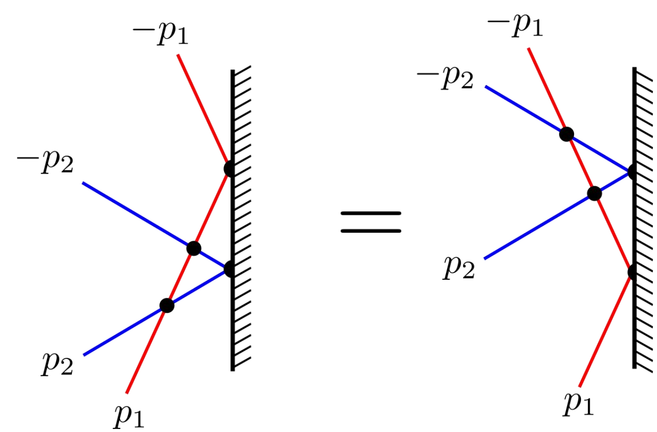
Boundary YBE



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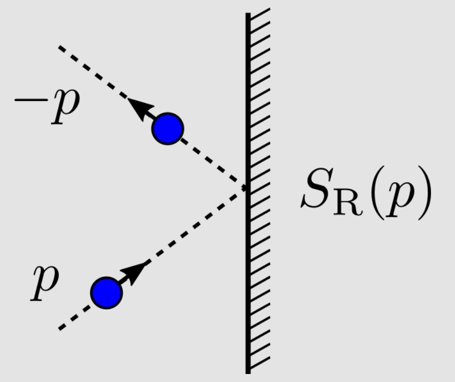

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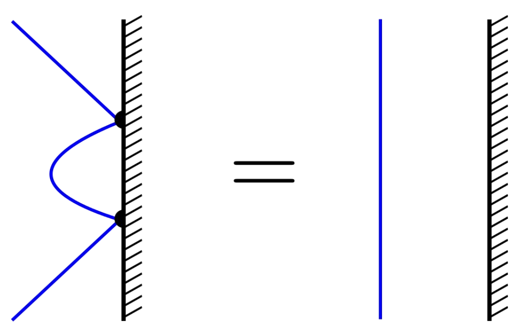
Boundary Unitarity



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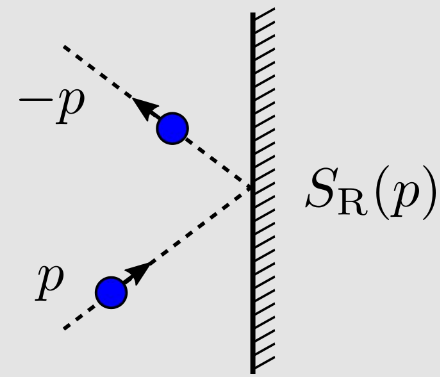
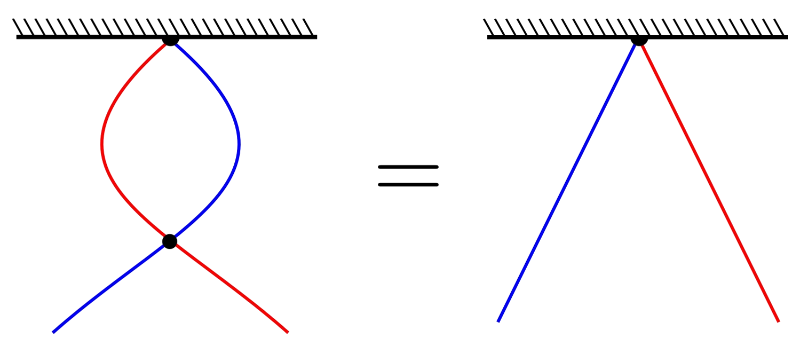


Boundary S-matrices can be determined by **bootstrap axioms**

Boundary Crossing



Boundary IQFT

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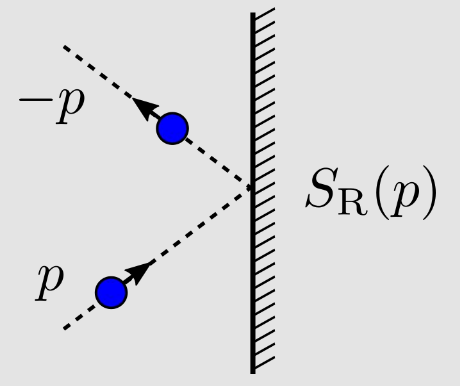


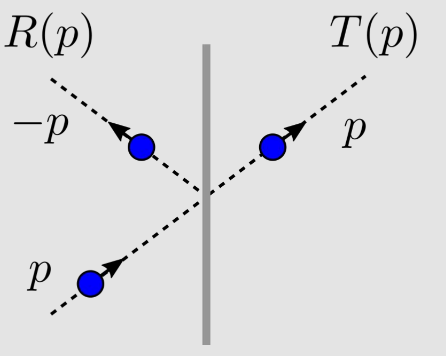
- Find boundary S-matrix by solving bootstrap axioms
- Compute other observables using integrability
- There are new observables, such as boundary free energy and exact *g*-function, boundary 1pt function, …

Defect IQFT

Bootstrap description

[Delfino, Mussardo and Simonetti 1994]

- Scattering can be transmissive and reflective
- Preserves integrability
- Transmission and reflection amplitudes



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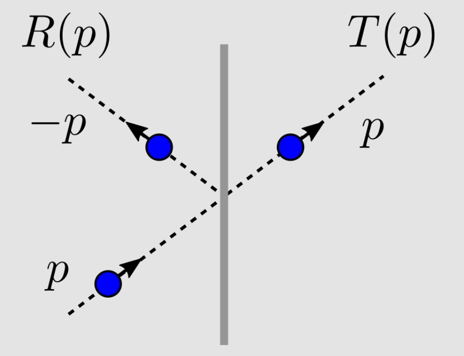

Topological vs non-topological

A purely transmissive is called *topological*, otherwise it is called *non-topological*

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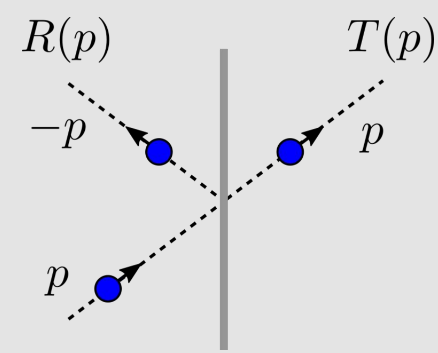


Topological vs non-topological

A purely transmissive is called *topological*, otherwise it is called *non-topological*

[Castro-Alvaredo, Fring and Gohmann 2002]

Theorem

Only free theories can have **integrable non-topological defects**

2. Solvable deformations

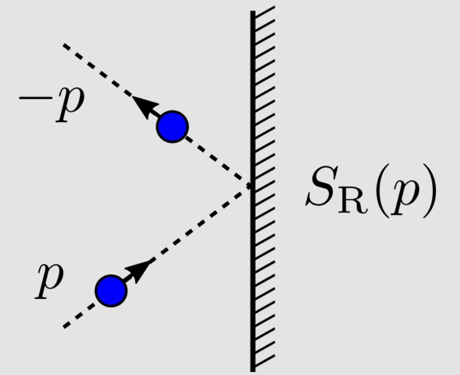
Deforming BIQFT

Even and odd charges

Integrability

$$\partial_{\bar{z}} T_{s+1} = \partial_z \Theta_{s-1}$$

$$\partial_z \bar{T}_{s+1} = \partial_{\bar{z}} \bar{\Theta}_{s-1}$$



Conserved charges

$$I_s = \int_{\mathcal{C}} \left(T_{s+1} \mathrm{d}z + \Theta_{s-1} \mathrm{d}\bar{z}\right) \qquad \bar{I}_s = \int_{\mathcal{C}} \left(\bar{T}_{s+1} \mathrm{d}\bar{z} + \bar{\Theta}_{s-1} \mathrm{d}z\right)$$

Define

$$H_s = I_s + \bar{I}_s \qquad P_s = -i(I_s - \bar{I}_s)$$

The *H*-type higher charges are preserved by the boundary

Deforming BIQFT

Bilinear deformations

Two currents

$$J^{\mu}_{H_s} = (\mathcal{H}_s, \mathcal{J}_{\mathcal{H}_s}) \qquad J^{\mu}_{P_r} = (\mathcal{P}_r, \mathcal{J}_{\mathcal{P}_r})$$

Bilinear deformation

$$\frac{\mathrm{d}}{\mathrm{d}\lambda} H_\lambda = \int_{s_{\mathrm{L}}}^{s_{\mathrm{R}}} \mathcal{O}_{rs}(x)\mathrm{d}x \qquad \mathcal{O}_{rs} = -\epsilon_{\mu\nu} J^{\mu}_{\mathcal{P}_r} J^{\nu}_{\mathcal{H}_s}$$

r=s=1 is the TTbar deformation

Deforming BIQFT

Bilinear deformations

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r=*s*=1 is the TTbar deformation

Bilocal deformations

[Bargheer, Beisert and Loebbert 2012]

$$\frac{\mathrm{d}}{\mathrm{d}\lambda} H_\lambda = [X_{rs}, H_\lambda] \qquad X_{rs} = \int_{x_1 < x_2} \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathcal{P}_r(x_1) \mathcal{H}_s(x_2)$$

[Kruthoff, Parrikar 2020] [Guica's talk]

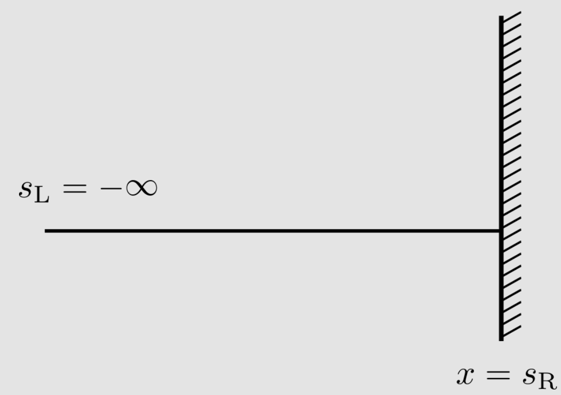
Deforming BIQFT

Relation between two deformations

$$[X_{rs}, H] = \int_{s_\mathrm{L}}^{s_\mathrm{R}} \mathcal{O}_{rs}(x)\mathrm{d}x - \mathcal{J}_{\mathcal{P}_r}(s_\mathrm{L})H_s + P_r\mathcal{J}_{\mathcal{H}_s}(s_\mathrm{R})$$

Half infinite line

$$\frac{\mathrm{d}H_\lambda}{\mathrm{d}\lambda} = [X_{rs}, H] = \int_{-\infty}^{s_\mathrm{R}} \mathcal{O}_{rs}(x)\mathrm{d}x$$



Deforming BIQFT

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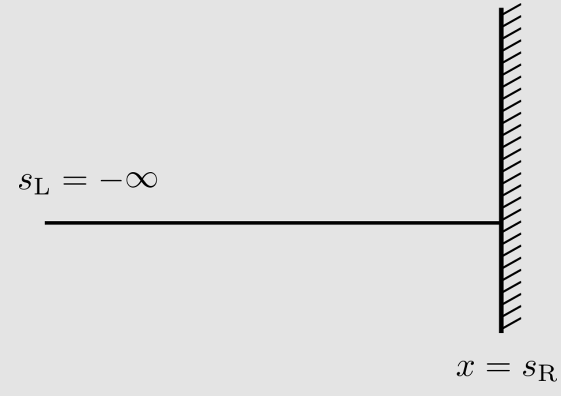
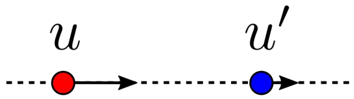


- For boundary case, order of two charges is important
- Use bi-local formulation and find deformed S-matrices

S-matrix bulk

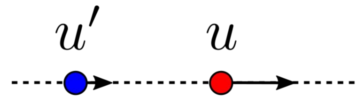
Asymptotic 2-particle S-matrix

$|u, u'\rangle =$



$|u < u'\rangle$

$+$



$|u' < u\rangle$

$+$ UV

local contribution

S-matrix bulk

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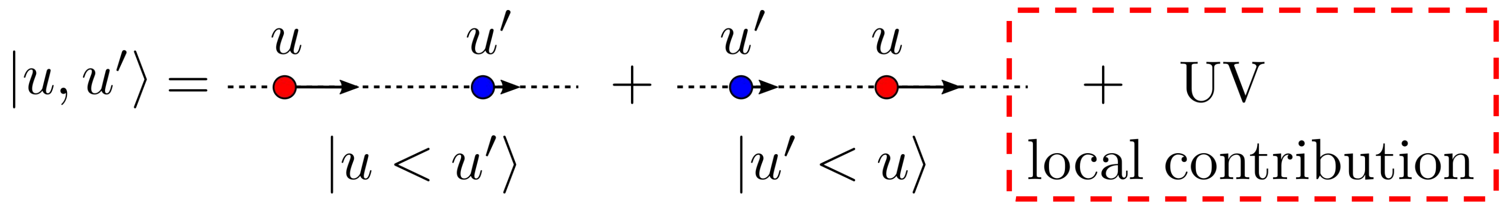

does not affect S-matrix

S-matrix bulk

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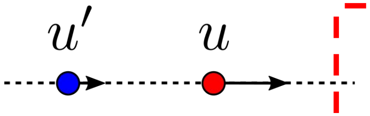
$|u,u'\rangle =$

$|u<u'\rangle$

$+$



$|u'<u\rangle$

$+$ UV local contribution

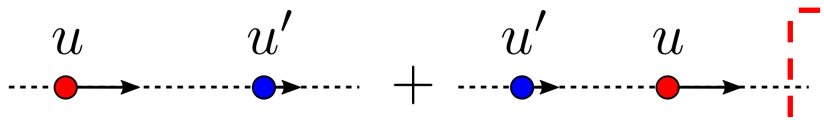
does not affect S-matrix

$$|u,u'\rangle \approx a(u,u')|u<u'\rangle + a(u',u)|u'<u\rangle$$

$$S(u,u') = \frac{a(u',u)}{a(u,u')}$$

S-matrix bulk

Asymptotic 2-particle S-matrix

$|u, u'\rangle =$  $+$ UV local contribution

$|u < u'\rangle$ $\quad$ $|u' < u\rangle$

does not affect S-matrix

$$|u, u'\rangle \approx a(u, u')|u < u'\rangle + a(u', u)|u' < u\rangle$$

$$S(u, u') = \frac{a(u', u)}{a(u, u')}$$

Deformed asymptotic state

$$|u, u'\rangle_\lambda \approx a_\lambda(u, u')|u < u'\rangle + a_\lambda(u', u)|u' < u\rangle$$

S-matrix bulk

Deformed S-matrix

$$|u, u'\rangle_\lambda \approx a_\lambda(u, u')|u < u'\rangle + a_\lambda(u', u)|u' < u\rangle$$

S-matrix bulk

Deformed S-matrix

$$|u,u'\rangle_\lambda \approx a_\lambda(u,u')|u<u'\rangle + a_\lambda(u',u)|u'<u\rangle$$

Starting from the eigenvalue equation

$$H_\lambda|u,u'\rangle_\lambda = [h(u)+h(u')]|u,u'\rangle_\lambda$$

S-matrix bulk

Deformed S-matrix

$$|u, u'\rangle_\lambda \approx a_\lambda(u, u')|u < u'\rangle + a_\lambda(u', u)|u' < u\rangle$$

Starting from the eigenvalue equation

$$H_\lambda|u, u'\rangle_\lambda = [h(u) + h(u')]|u, u'\rangle_\lambda$$

Taking derivative

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\big([H_\lambda - h(u) - h(u')]|u, u'\rangle_\lambda\big) = 0$$

S-matrix bulk

Deformed S-matrix

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$$\frac{\mathrm{d}}{\mathrm{d}\lambda}H_\lambda = [X_{rs}, H_\lambda]$$

S-matrix bulk

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Find that

$$X_{rs}|u, u'\rangle_\lambda = \frac{\mathrm{d}a_\lambda(u, u')}{\mathrm{d}\lambda}|u < u'\rangle + \frac{\mathrm{d}a_\lambda(u', u)}{\mathrm{d}\lambda}|u' < u\rangle$$

S-matrix bulk

Deformed S-matrix

$$X_{rs}|u, u'\rangle_\lambda = \frac{\mathrm{d}a_\lambda(u, u')}{\mathrm{d}\lambda}|u < u'\rangle + \frac{\mathrm{d}a_\lambda(u', u)}{\mathrm{d}\lambda}|u' < u\rangle$$

S-matrix bulk

Deformed S-matrix

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Using the fact

$$X_{rs}|u<u'\rangle = \left[ip_r(u)h_s(u') + f_{rs}(u) + f_{rs}(u')\right]|u<u'\rangle$$

S-matrix bulk

Deformed S-matrix

$$X_{rs}|u,u'\rangle_\lambda = \frac{\mathrm{d}a_\lambda(u,u')}{\mathrm{d}\lambda}|u<u'\rangle + \frac{\mathrm{d}a_\lambda(u',u)}{\mathrm{d}\lambda}|u'<u\rangle$$

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$$X_{rs} = \int_{x_1<x_2} \mathrm{d}x_1\,\mathrm{d}x_2\,\mathcal{P}_r(x_1)\mathcal{H}_s(x_2)$$

S-matrix bulk

Deformed S-matrix

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$$X_{rs}|u<u'\rangle = [ip_r(u)h_s(u') + f_{rs}(u) + f_{rs}(u')]\,|u<u'\rangle$$

$$X_{rs} = \int_{x_1<x_2} \mathrm{d}x_1\,\mathrm{d}x_2\,\mathcal{P}_r(x_1)\mathcal{H}_s(x_2)$$

both density act on the same particle

S-matrix bulk

Deformed S-matrix

$$X_{rs}|u,u'\rangle_\lambda = \frac{\mathrm{d}a_\lambda(u,u')}{\mathrm{d}\lambda}|u<u'\rangle + \frac{\mathrm{d}a_\lambda(u',u)}{\mathrm{d}\lambda}|u'<u\rangle$$

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S-matrix bulk

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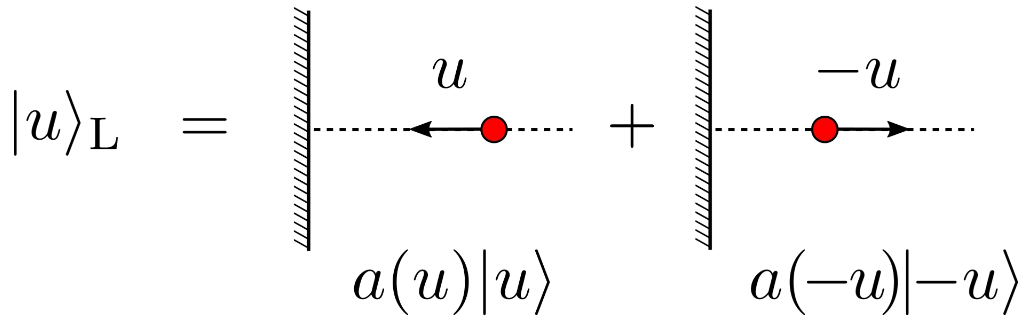
$$X_{rs}|u'<u\rangle = [ip_r(u')h_s(u) + f_{rs}(u') + f_{rs}(u)]\,|u'<u\rangle$$

We find that [Smirnov and Zamolodchikov 2016]

$$S_\lambda(u,u') = e^{-i\lambda(p_r(u)h_s(u')-p_r(u')h_s(u))}S(u,u')$$

S-matrix boundary

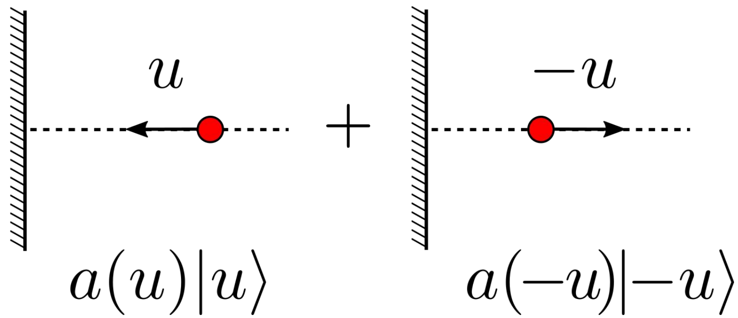
Boundary asymptotic state



S-matrix boundary

Boundary asymptotic state

$|u\rangle_{\mathrm{L}} =$

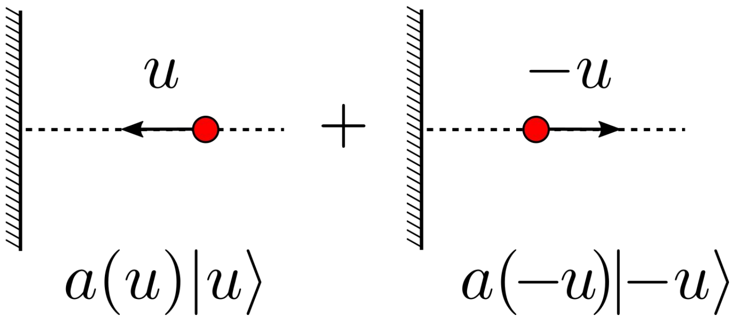


$$S_{\mathrm{L}}(u) = \frac{a(u)}{a(-u)}$$

S-matrix boundary

Boundary asymptotic state

$|u\rangle_{\mathrm{L}} =$



$$S_{\mathrm{L}}(u) = \frac{a(u)}{a(-u)}$$

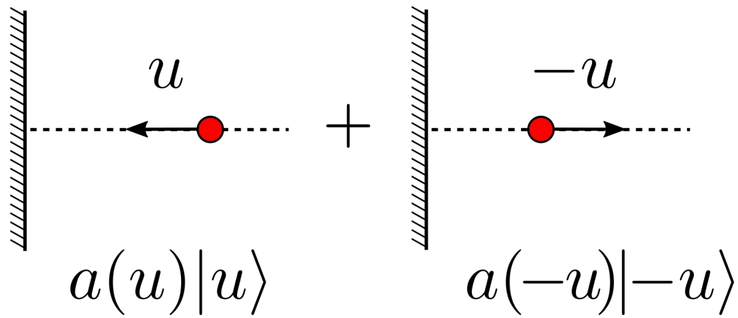
Deformed asymptotic state

$$|u\rangle_{\mathrm{L},\lambda} = a_\lambda(u)|u\rangle + a_\lambda(-u)| - u\rangle$$

S-matrix boundary

Boundary asymptotic state

$|u\rangle_{\mathrm{L}} =$



$$S_{\mathrm{L}}(u) = \frac{a(u)}{a(-u)}$$

Deformed asymptotic state

$$|u\rangle_{\mathrm{L},\lambda} = a_\lambda(u)|u\rangle + a_\lambda(-u)| - u\rangle$$

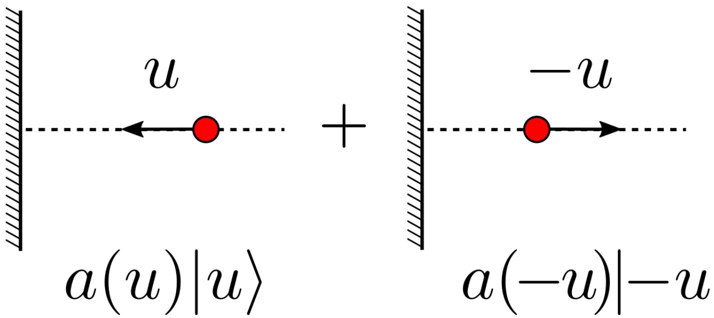
Taking derivative of eigenvalue equation

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\Big([H_\lambda - h(u)]|u\rangle_{\mathrm{L},\lambda}\Big) = 0$$

S-matrix boundary

Boundary asymptotic state

$|u\rangle_{\mathrm{L}} =$



$$S_{\mathrm{L}}(u) = \frac{a(u)}{a(-u)}$$

Deformed asymptotic state

$$|u\rangle_{\mathrm{L},\lambda} = a_\lambda(u)|u\rangle + a_\lambda(-u)| - u\rangle$$

We obtain

$$X|u\rangle_{\mathrm{L},\lambda} = \frac{\mathrm{d}a_\lambda(u)}{\mathrm{d}\lambda}|u\rangle + \frac{\mathrm{d}a_\lambda(-u)}{\mathrm{d}\lambda}|-u\rangle$$

S-matrix boundary

Different cases

Bilocal deformation $X = [H_r | P_s]$

$$S_{\mathrm{L},\lambda}(u) = e^{i\lambda h_r(u) p_s(u)} S_{\mathrm{L}}(u)$$

Confirmed earlier proposal in CFT [Caselle, Fioravanti, Gliozzi and Tateo 2013]

S-matrix boundary

Different cases

Bilocal deformation $X = [H_r | P_s]$

$$S_{\mathrm{L},\lambda}(u) = e^{i\lambda h_r(u) p_s(u)} S_{\mathrm{L}}(u)$$

Confirmed earlier proposal in CFT [Caselle, Fioravanti, Gliozzi and Tateo 2013]

Odd charges $X = P_r$ [Loebbert 2012]

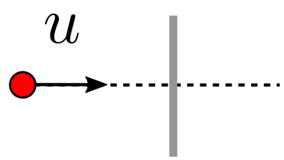
$$S_{\mathrm{L},\lambda}(u) = e^{2i\lambda p_r(u)} S_{\mathrm{L}}(u)$$

Specific for the boundary case. **Does not change bulk S-matrix**, only change the boundary S-matrix.

S-matrix defect

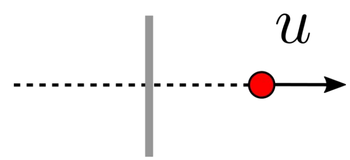
Topological defect

$$|u\rangle_{\mathrm{D}} =$$



$$a(u)|u;\varnothing\rangle$$

$$+$$



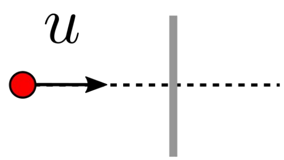
$$b(u)|\varnothing;u\rangle$$

$$T(u) = \frac{b(u)}{a(u)}$$

S-matrix defect

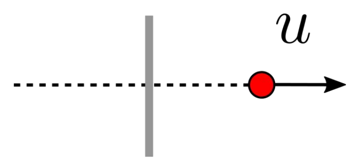
Topological defect

$|u\rangle_{\mathrm{D}} =$



$a(u)|u;\varnothing\rangle$

$+$



$b(u)|\varnothing;u\rangle$

$$T(u) = \frac{b(u)}{a(u)}$$

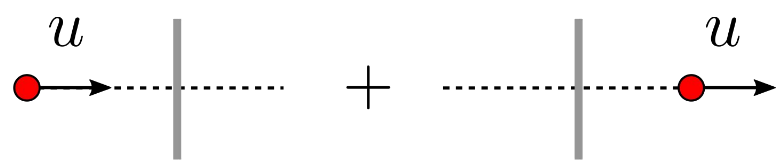
Deformed asymptotic state

$$|u\rangle_{\mathrm{D},\lambda} = a_\lambda(u)|u;\varnothing\rangle + b_\lambda(u)|\varnothing;u\rangle$$

S-matrix defect

Topological defect

$$|u\rangle_{\mathrm{D}} = a(u)|u;\varnothing\rangle + b(u)|\varnothing;u\rangle$$



$a(u)|u;\varnothing\rangle$ $\qquad$ $b(u)|\varnothing;u\rangle$

$$T(u) = \frac{b(u)}{a(u)}$$

Deformed asymptotic state

$$|u\rangle_{\mathrm{D},\lambda} = a_\lambda(u)|u;\varnothing\rangle + b_\lambda(u)|\varnothing;u\rangle$$

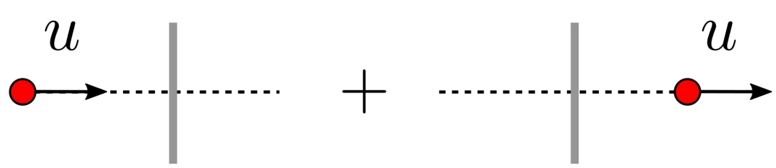
Taking derivative

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\Big([H_\lambda - h(u)]|u\rangle_{\mathrm{D},\lambda}\Big) = 0$$

S-matrix defect

Topological defect

$|u\rangle_{\rm D} =$



$a(u)|u;\varnothing\rangle \qquad b(u)|\varnothing;u\rangle$

$$T(u) = \frac{b(u)}{a(u)}$$

Deformed asymptotic state

$$|u\rangle_{\rm D,\lambda} = a_\lambda(u)|u;\varnothing\rangle + b_\lambda(u)|\varnothing;u\rangle$$

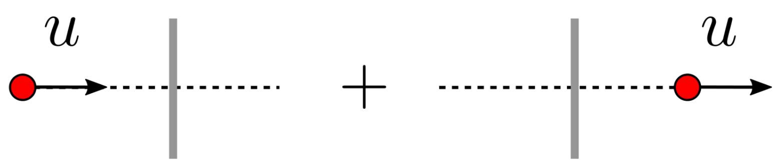
We find that

$$X|u\rangle_{\rm D,\lambda} = \frac{{\rm d}a_\lambda(u)}{{\rm d}\lambda}|u;\varnothing\rangle + \frac{{\rm d}b_\lambda(u)}{{\rm d}\lambda}|\varnothing;u\rangle$$

S-matrix defect

Topological defect

$$|u\rangle_{\mathrm{D}} = $$



$$a(u)|u;\varnothing\rangle \qquad b(u)|\varnothing;u\rangle$$

$$T(u) = \frac{b(u)}{a(u)}$$

Deformed asymptotic state

$$|u\rangle_{\mathrm{D},\lambda} = a_\lambda(u)|u;\varnothing\rangle + b_\lambda(u)|\varnothing;u\rangle$$

This leads to

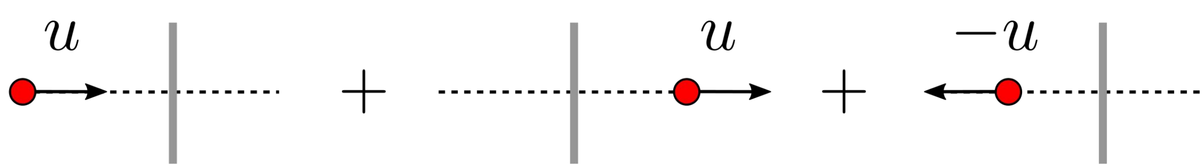
$$T_\lambda(u) = \frac{b_\lambda(u)}{a_\lambda(u)} = \frac{b(u)}{a(u)} = T(u)$$

The **topological defect** is **not affected** !

S-matrix defect

Non-topological defect

$|u\rangle_{\mathrm{D}} \; =$



$$a(u)|u;\varnothing\rangle \qquad b(u)|\varnothing;u\rangle \qquad c(u)|-u;\varnothing\rangle$$

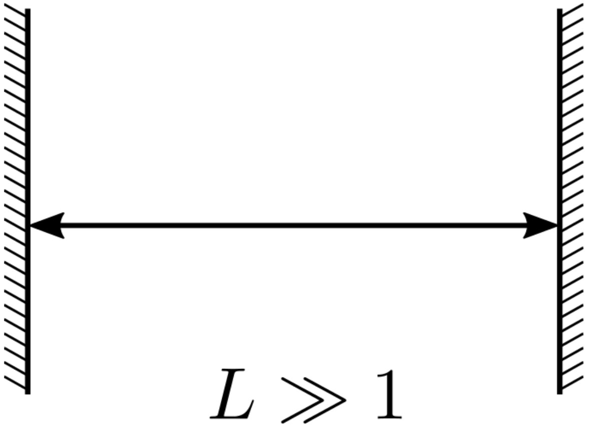
$$T(u) = \frac{b(u)}{a(u)} \qquad\qquad R(u) = \frac{c(u)}{a(u)}$$

- The reflection amplitude deformed like the boundary S-matrix
- The transmission amplitude not deformed

3. Deformed observables

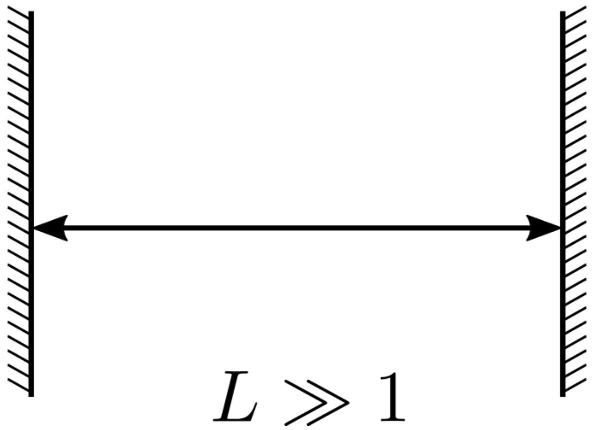
Deformed spectrum I

Large volume limit



Deformed spectrum I

Large volume limit



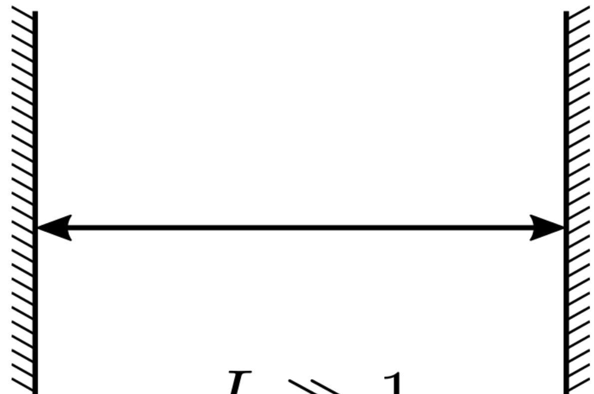
Asymptotic Bethe ansatz equation

$$e^{2ip(u_j)L} S_{\mathrm{L}}(u_j) S_{\mathrm{R}}(-u_j) \prod_{k \neq j}^{N} S(u_j, u_k) S(u_j, -u_k) = 1$$

Quantization condition for an *N* particle state

Deformed spectrum I

Large volume limit



$$L \gg 1$$

Spectrum

$$E_N = \sum_{j=1}^{N} h(u_j)$$

Asymptotic Bethe ansatz equation

$$e^{2ip(u_j)L} S_{\mathrm{L}}(u_j) S_{\mathrm{R}}(-u_j) \prod_{k \neq j}^{N} S(u_j, u_k) S(u_j, -u_k) = 1$$

Quantization condition for an *N* particle state

Deformed spectrum I bilocal deformation

Take the deformation $X_{rs} = [P_r | H_s]$

Deformed S-matrices

$$S_\lambda(u,v) = S(u,v)\, e^{-i\lambda(p_r(u)h_s(v) - h_s(u)p_r(v))}$$

$$S_{\mathrm{L},\lambda}(u) = S_{\mathrm{L}}(u)\, e^{i\lambda p_r(u)h_s(u)}$$

$$S_{\mathrm{R},\lambda}(u) = S_{\mathrm{R}}(u)\, e^{-i\lambda p_r(u)h_s(u)}$$

$$p_r(u) = \gamma_r \sinh(ru) \qquad h_s(u) = \gamma_s \cosh(su)$$

Deformed spectrum I bilocal deformation

Take the deformation $X_{rs} = [P_r | H_s]$

Deformed S-matrices

$$S_\lambda(u,v) = S(u,v)\, e^{-i\lambda(p_r(u)h_s(v) - h_s(u)p_r(v))}$$

$$S_{\mathrm{L},\lambda}(u) = S_{\mathrm{L}}(u)\, e^{i\lambda p_r(u)h_s(u)}$$

$$S_{\mathrm{R},\lambda}(u) = S_{\mathrm{R}}(u)\, e^{-i\lambda p_r(u)h_s(u)}$$

- For *r*=*s*, Lorentz invariance preserved, CDD factors
- For *r*=1, change effective length, dynamical hard rod picture

Deformed spectrum I bilocal deformation

Take r=1

$$X_s = [P|H_s]$$

Deformed spectrum I bilocal deformation

Take r=1 $$X_s = [P|H_s]$$

Deformed Bethe equation

$$e^{2iLp(u_j)} S_{\mathrm{L}}(u_j) S_{\mathrm{R}}(-u_j) \prod_{k\neq j}^{N} S(u_j, u_k) S(u_j, -u_k) = e^{-2i\lambda Q_N^{(s)} p(u_j)}$$

Deformed spectrum I bilocal deformation

Take *r*=1

$$X_s = [P|H_s]$$

$$Q_N^{(s)} = \sum_{k=1}^{N} h_s(u_j)$$

Deformed Bethe equation

$$e^{2iLp(u_j)} S_{\rm L}(u_j) S_{\rm R}(-u_j) \prod_{k\neq j}^{N} S(u_j, u_k) S(u_j, -u_k) = e^{-2i\lambda Q_N^{(s)} p(u_j)}$$

Deformed spectrum I bilocal deformation

Take r=1

$$X_s = [P|H_s]$$

$$Q_N^{(s)} = \sum_{k=1}^{N} h_s(u_j)$$

Deformed Bethe equation

$$e^{2iLp(u_j)} S_{\mathrm{L}}(u_j) S_{\mathrm{R}}(-u_j) \prod_{k\neq j}^{N} S(u_j, u_k) S(u_j, -u_k) = e^{-2i\lambda Q_N^{(s)} p(u_j)}$$

Equivalent to the original BAE with

$$L \to L + \lambda\, Q_N^{(s)}$$

Deformed spectrum I bilocal deformation

Take r=1

$$X_s = [P|H_s]$$

$$Q_N^{(s)} = \sum_{k=1}^{N} h_s(u_j)$$

Deformed Bethe equation

$$e^{2iLp(u_j)} S_{\rm L}(u_j) S_{\rm R}(-u_j) \prod_{k\neq j}^{N} S(u_j,u_k) S(u_j,-u_k) = e^{-2i\lambda Q_N^{(s)} p(u_j)}$$

This leads to the flow equation

$$\partial_\lambda E_N(\lambda, L) = Q_N^{(s)} \partial_L E_N(\lambda, L)$$

Deformed spectrum I bilocal deformation

Take r=1

$$X_s = [P|H_s]$$

$$Q_N^{(s)} = \sum_{k=1}^{N} h_s(u_j)$$

Deformed Bethe equation

$$e^{2iLp(u_j)} S_{\mathrm{L}}(u_j) S_{\mathrm{R}}(-u_j) \prod_{k \neq j}^{N} S(u_j, u_k) S(u_j, -u_k) = e^{-2i\lambda Q_N^{(s)} p(u_j)}$$

This leads to the flow equation

$$\partial_\lambda E_N(\lambda, L) = Q_N^{(s)} \partial_L E_N(\lambda, L)$$

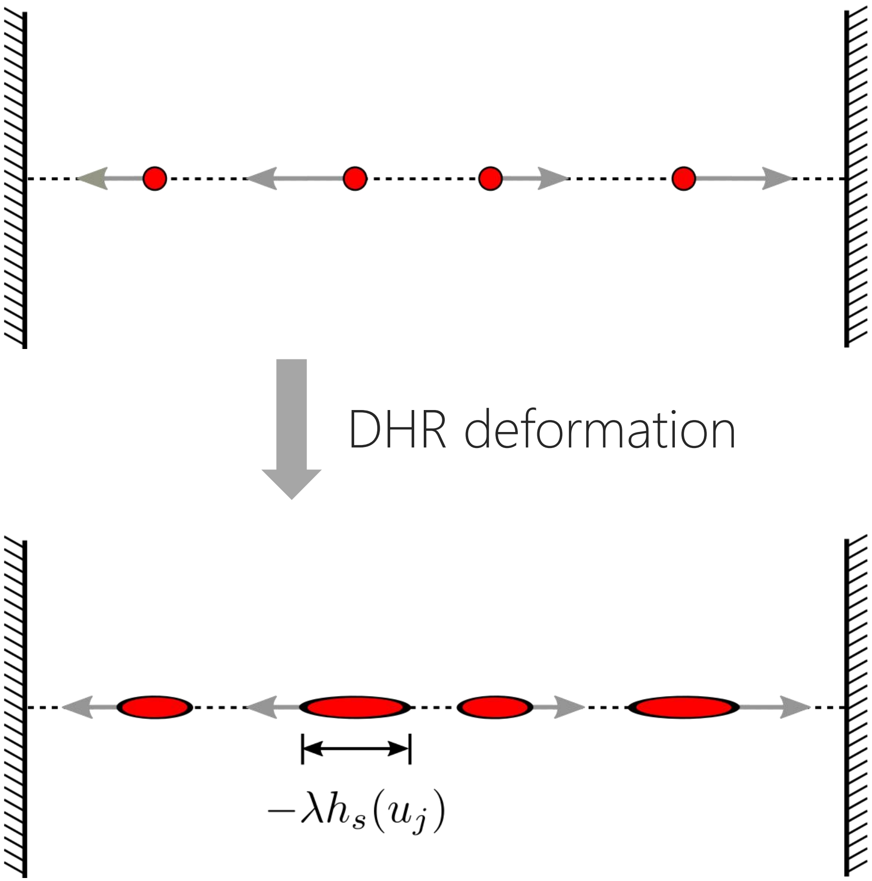
The effective length is changed, this can be interpreted by the dynamical hard rod picture.

Deformed spectrum I bilocal deformation

Dynamical hard rod

[Cardy and Doyon 2020]

[YJ 2020]



$$L \to L + \lambda\, Q_N^{(s)}$$

- Point particles becomes finite length hard rods
- Length of each rod proportional to its charge
- For the other sign, distance between particles are increased

Deformed spectrum I bilocal deformation

More general case

Consider more general BAE $X_{rs} = [P_r|H_s]$

$$e^{2iL[p(u_j)+\nu_r\, p_r(u_j)]} S_{\rm L}(u_j) S_{\rm R}(-u_j) \prod_{k\neq j}^{N} S(u_j,u_k) S(u_j,-u_k) = 1$$

Deformed spectrum I bilocal deformation

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a new twist

Deformed spectrum I bilocal deformation

More general case

Consider more general BAE $X_{rs} = [P_r | H_s]$

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$$e^{2iL[p(u_j)+\nu_r\, p_r(u_j)]} S_{\rm L}(u_j) S_{\rm R}(-u_j) \prod_{k\neq j}^{N} S(u_j,u_k) S(u_j,-u_k) = e^{-2i\lambda Q_N^{(s)} p_r(u_j)}$$

Effectively changes chemical potential

$$\nu_r \to \nu_r + \frac{\lambda Q_N^{(s)}}{L}$$

Deformed spectrum I bilocal deformation

More general case

Consider more general BAE $X_{rs} = [P_r | H_s]$

$$e^{2iL[p(u_j)+\nu_r\, p_r(u_j)]} S_{\rm L}(u_j) S_{\rm R}(-u_j) \prod_{k\neq j}^{N} S(u_j,u_k) S(u_j,-u_k) = 1$$

a new twist

$$e^{2iL[p(u_j)+\nu_r\, p_r(u_j)]} S_{\rm L}(u_j) S_{\rm R}(-u_j) \prod_{k\neq j}^{N} S(u_j,u_k) S(u_j,-u_k) = e^{-2i\lambda Q_N^{(s)} p_r(u_j)}$$

Flow equation for the spectrum

$$\partial_\lambda E_N(\lambda, L, \nu_r) = \frac{1}{L} Q_N^{(s)} \partial_{\nu_r} E_N(\lambda, L, \nu_r)$$

Deformed spectrum I odd charge

Take the deformation $X = P_r$

Deformed S-matrices

$$S_\lambda(u,v) = S(u,v)$$

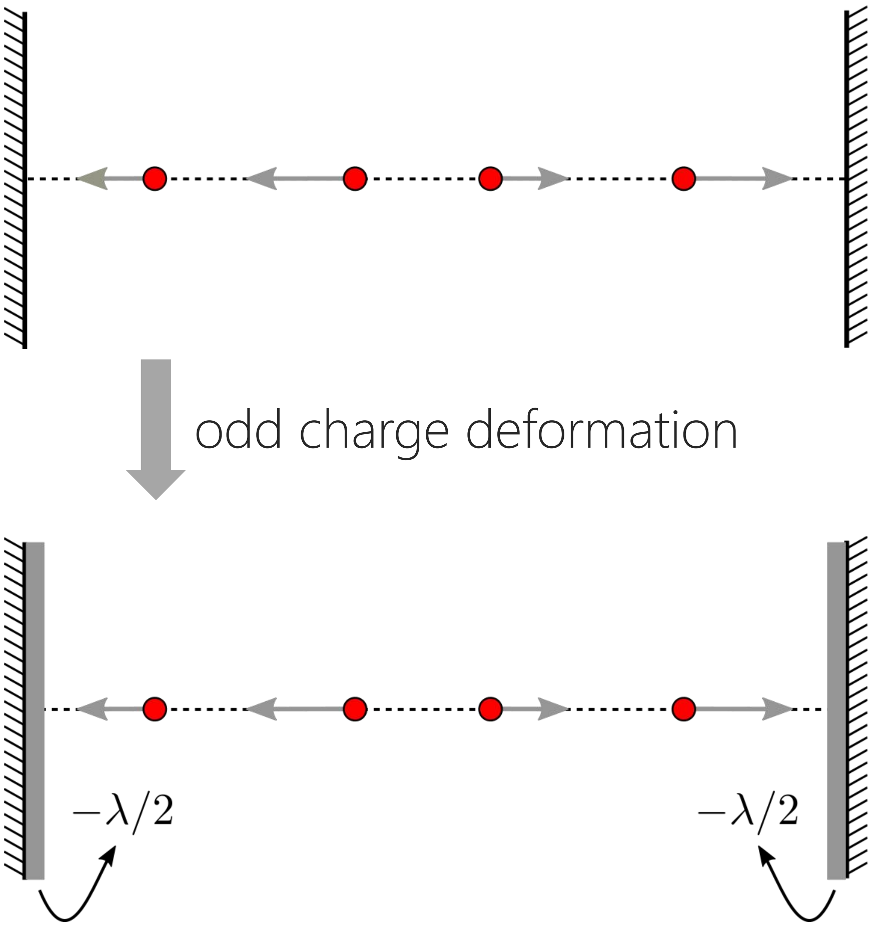
$$S_{\mathrm{L},\lambda}(u) = S_{\mathrm{L}}(u)\, e^{i\lambda p_r(u)}$$

$$S_{\mathrm{R},\lambda}(u) = S_{\mathrm{R}}(u)\, e^{-i\lambda p_r(u)}$$

- These deformation do not change bulk S-matrix
- For *r*=1, change effective length

Deformed spectrum I odd charge

A thick wall



$$L \to L + \lambda$$

- This is specific to the boundary case with r=1
- The boundaries become “thicker”
- For the other sign, distance between boundaries are increased

Deformed spectrum I odd charge

Flow equation for spectrum

For r=1

$$\partial_\lambda E_N(\lambda, L) = \partial_L E_N(\lambda, L)$$

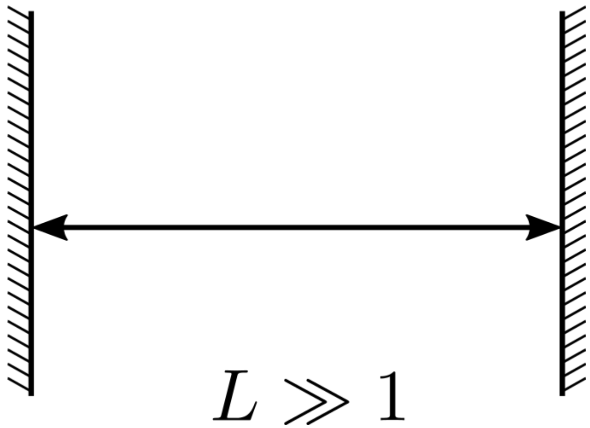
For r>1

$$\partial_\lambda E_N(\lambda, L, \nu_r) = \frac{1}{L}\partial_{\nu_r} E_N(\lambda, L, \nu_r)$$

- These are linear equations instead of non-linear ones
- Do not depend on details of the bulk excitations

Deformed spectrum I

Summary



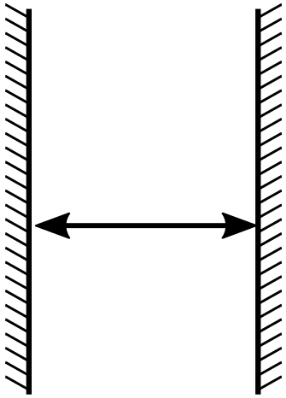
- For large volume, deformed spectrum can be obtained from **boundary BAE**

- We obtained **simple flow equation** for finite volume spectrum. They are non-linear (linear) for bi-local and linear for (odd charge) deformations.

- Typically such flow equations are robust and do not depend on the volume. They should also **hold in finite volume**.

Deformed spectrum II

Finite volume

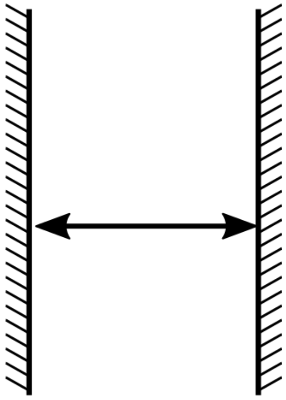


- For finite volume, **finite size corrections** become important.
- We apply the **boundary Thermodynamic Bethe ansatz** to compute the spectrum
- We can **verify the flow equation** which we obtained in the large volume limit

[LeClair, Mussardo, Saleur and Skorik 1995]

Deformed spectrum II

Finite volume



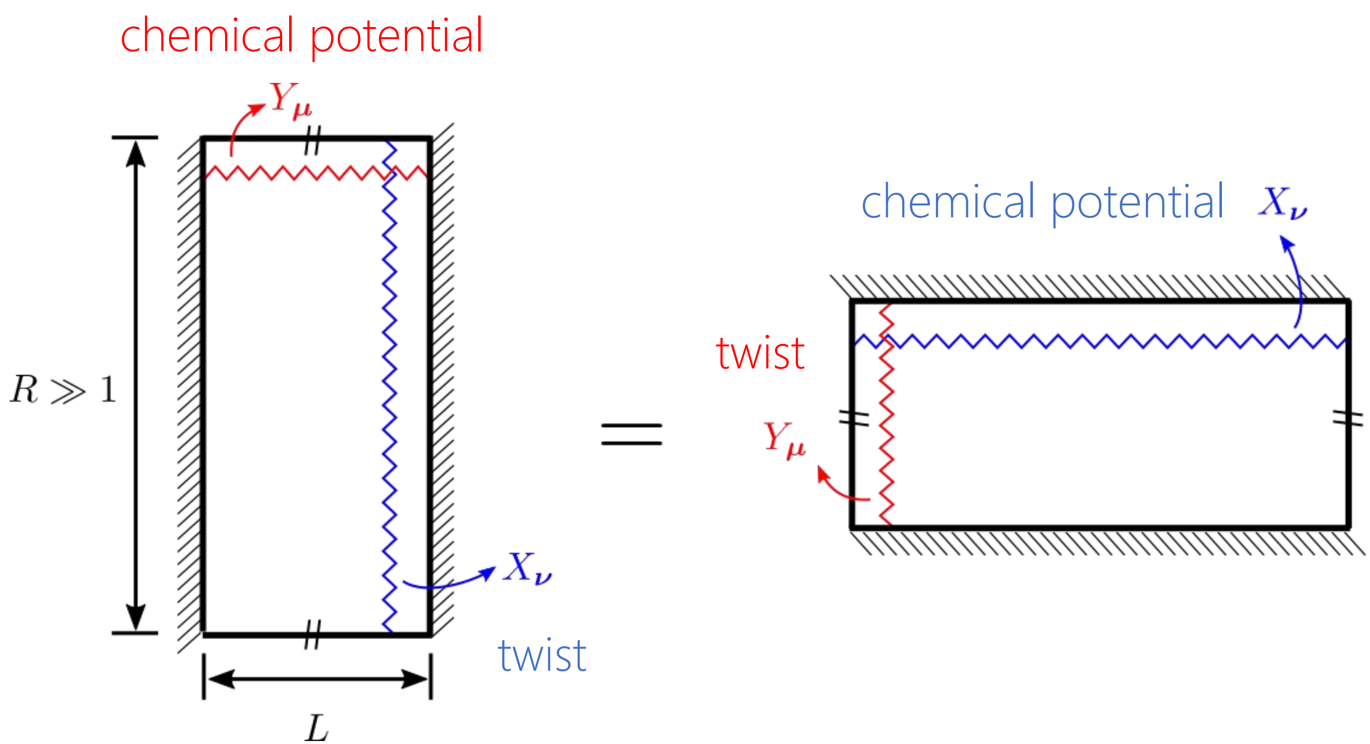
- For finite volume, **finite size corrections** become important.
- We apply the **boundary Thermodynamic Bethe ansatz** to compute the spectrum
- We can **verify the flow equation** which we obtained in the large volume limit

[LeClair, Mussardo, Saleur and Skorik 1995]

- General bilocal deformation involve **higher charges**
- Accordingly, we introduce **twists** in Bethe equation
- Related to **chemical potentials** in the mirror channel

Deformed spectrum II bilocal deformation

Boundary TBA $X_{rs} = [P_r|H_s]$



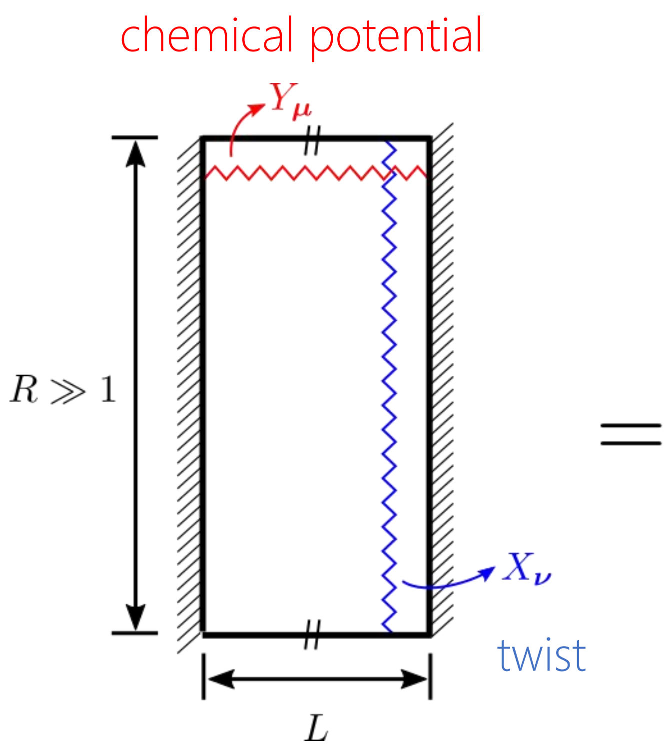
[Hernandez-Chifflet, Negro and Sfondrini 2019]

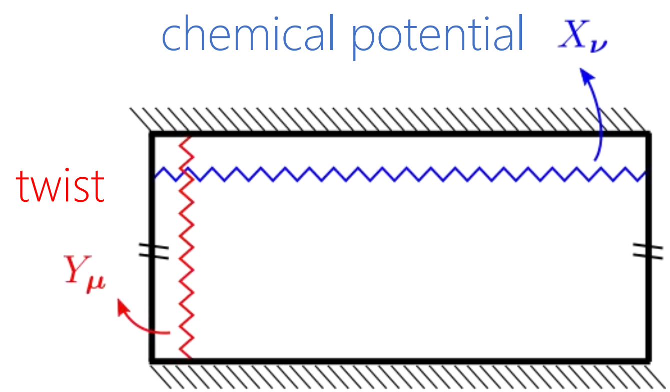
Deformed spectrum II bilocal deformation

Boundary TBA $X_{rs} = [P_r | H_s]$

Double Wick Rotation

$P \mapsto i\widetilde{H} \qquad H \mapsto i\widetilde{P}$





[Hernandez-Chifflet, Negro and Sfondrini 2019]

Deformed spectrum II bilocal deformation

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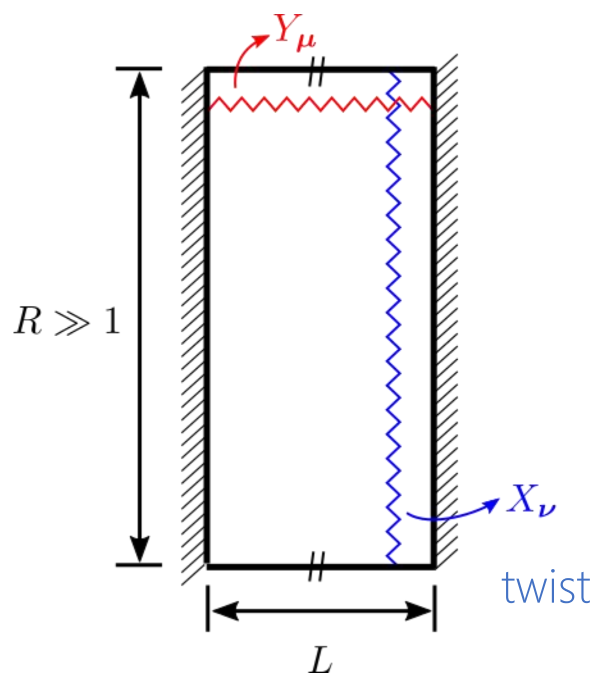

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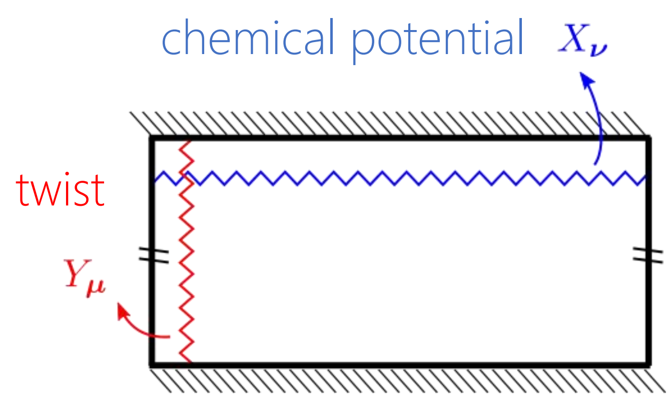


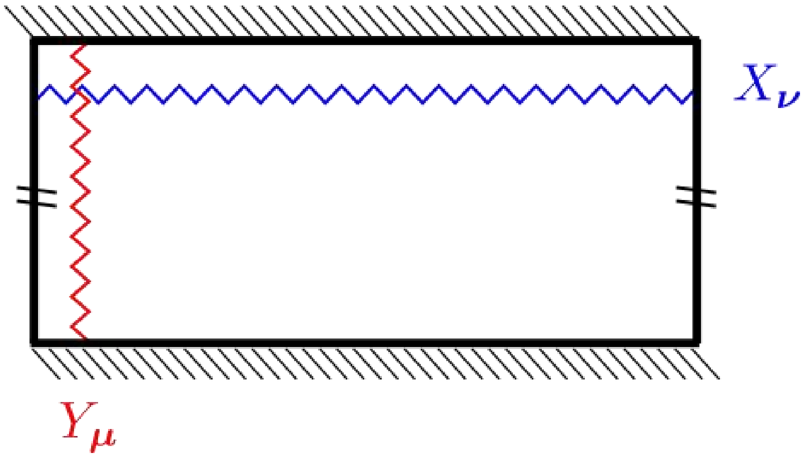
$Z_{ab} \sim \mathrm{tr}\left[e^{-R(H+\mu_s H_s)}\right]$

$Z_{ab} \sim \langle B_a|e^{-L(\widetilde{H}+\nu_r \widetilde{H}_r)}|B_b\rangle$

[Hernandez-Chifflet, Negro and Sfondrini 2019]

Deformed spectrum II bilocal deformation

Boundary TBA



Compute partition function in the closed channel.

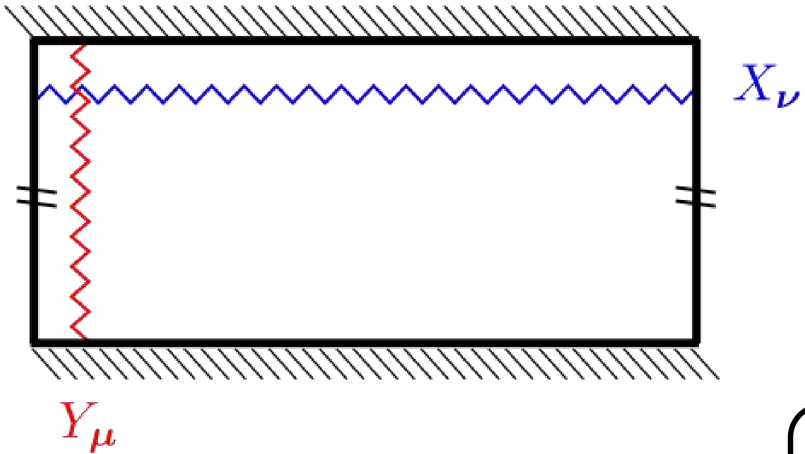
$$Z_{ab} \sim \langle B_a | e^{-L(\widetilde{H} + \nu_r \widetilde{H}_r)} | B_b \rangle$$

Quantization condition

$$e^{RY_\mu\left(u+\frac{i\pi}{2}\right)} \widetilde{S}(u_j, -u_j) \prod_{k\neq j}^{N} \widetilde{S}(u_j, u_k) \widetilde{S}(u_j, -u_k) = 1$$

Deformed spectrum II bilocal deformation

Boundary TBA



Compute partition function in the closed channel.

$$Z_{ab} \sim \langle B_a | e^{-L(\widetilde{H} + \nu_r \widetilde{H}_r)} | B_b \rangle$$

Quantization condition

$$Y_\mu(u) = e(u) + \mu_s e_s(u)$$

$$e^{RY_\mu\left(u+\frac{i\pi}{2}\right)} \widetilde{S}(u_j, -u_j) \prod_{k \neq j}^{N} \widetilde{S}(u_j, u_k) \widetilde{S}(u_j, -u_k) = 1$$

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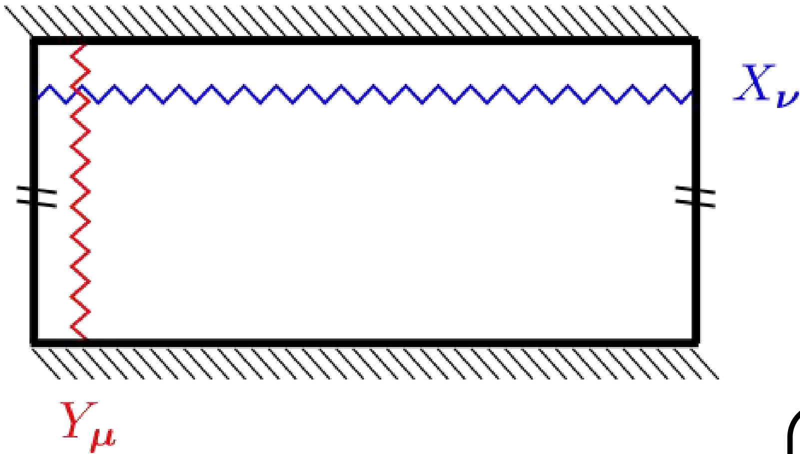

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S-matrix in the mirror channel

Deformed spectrum II bilocal deformation

Boundary TBA

Following standard procedure

$$\epsilon(u) = 2LX_\nu(u) - \log\left[\chi_{ab}(u)\right] - \log\left(1 + e^{-\epsilon}\right) \star \tilde{\varphi}_+$$

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$$X_\nu(u) = h(u) + \nu_r\, h_r(u)$$

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$$X_\nu(u) = h(u) + \nu_r\, h_r(u)$$

$$\chi_{ab}(u) = \overline{K}_a(u) K_b(u)$$

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$$E^{(0)}(L,\nu_r) = -\frac{1}{2\pi i}\int_0^\infty \partial_u h\left(u+\tfrac{i\pi}{2}\right)\log\left(1+e^{-\epsilon}\right)\mathrm{d}u$$

$$Q_s^{(0)}(L,\nu_r) = -\frac{1}{2\pi i}\int_0^\infty \partial_u h_s\left(u+\tfrac{i\pi}{2}\right)\log\left(1+e^{-\epsilon}\right)\mathrm{d}u$$

Deformed spectrum II bilocal deformation

Boundary TBA

Deformed BTBA equation

$$\epsilon_\lambda(u) = 2LX_\nu(u) - \log\left[\chi_{ab}(u)\right] - \log\left(1 + e^{-\epsilon_\lambda}\right) \star \tilde{\varphi}_{+,\lambda}$$

Deformed spectrum II bilocal deformation

Boundary TBA

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↓

$$\epsilon_\lambda(u) = 2L\widehat{X}_\nu(u) - \log\left[\chi_{ab}(u)\right] - \log\left(1 + e^{-\epsilon_\lambda}\right) \star \tilde{\varphi}_{+}$$

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$$\nu_r \to \nu_r + \frac{\lambda}{L} Q_s^{(0)}$$

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$$\nu_r \to \nu_r + \frac{\lambda}{L}Q_s^{(0)}$$

For r=1

$$L \to L + \lambda Q_s^{(0)}$$

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Flow equation for $r>1$

$$\partial_\lambda E^{(0)}(\lambda, L, \nu_r) = \frac{1}{L} Q^{(0)}_{s,\lambda} \partial_{\nu_r} E^{(0)}(\lambda, L, \nu_r)$$

Flow equation for $r=1$

$$\partial_\lambda E^{(0)}(\lambda, L) = Q^{(0)}_{s,\lambda} \partial_L E^{(0)}(\lambda, L)$$

- These are the flow equation for ground state
- Matches exactly the one with large volume limit

Further comments

TTbar deformed partition function

Flow equation of the **partition function**

$$\partial_\lambda Z_{ab}(R,L|\lambda) = -\left(\frac{\partial}{\partial R} - \frac{1}{R}\right)\partial_L Z_{ab}(R,L|\lambda)$$

- This is the same as the one derived from random geometry [Cardy 2018]
- For other bilocal deformations, it seems hard to write down similar flow equation
- We can also write down the flow equation for the boundary entropy, or the exact *g*-function.

Conclusions

We studied solvable irrelevant deformations for IQFTs with **integrable boundaries and defects**

The **deformed S-matrix** can be determined. Deformed spectrum follows by standard methods

The integrable boundary has a specific solvable deformation which involve only the **odd charges**

The **topological defects** are **not deformed** by bilocal deformations

Outlook

- General boundary

  What about boundaries that are not necessarily integrable

- Other observables

  Defect and boundary correlation functions

- Other theories

  Deformed CFT boundary states, Bose gas with boundary