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Integrability and chaos in SYM theories from anomalous-dimension spectra

arXiv:2011.04633 & ongoing work
with Tristan McLoughlin & Raul Pereira

Exact results on irrelevant deformations of QFTs, APCTP 2021

Anne Spiering (Trinity College Dublin & Niels Bohr Institute)

Overview

Overview

1.1 Operator mixing in $\mathcal{N}=4$ SYM theory

$\mathfrak{su}(2)$ sector of $SU(N)$ $\mathcal{N}=4$ SYM theory

$$\mathcal{D} = :\mathrm{Tr}(Z\check{Z} + X\check{X}): - \frac{2\lambda}{N} :\mathrm{Tr}([X,Z][\check{X},\check{Z}]): + \mathcal{O}(\lambda^2)$$

$$\mathrm{Tr}(A\check{Z})\mathrm{Tr}(BZ) = \mathrm{Tr}(AB) - N^{-1}\mathrm{Tr}(A)\mathrm{Tr}(B)$$
$$\mathrm{Tr}(A\check{Z}BZ) = \mathrm{Tr}(A)\mathrm{Tr}(B) - N^{-1}\mathrm{Tr}(AB)$$

Example: mixing of operators $\mathrm{tr}(ZZXX),\ \mathrm{tr}(ZXZX),\ \mathrm{tr}(ZZ)\mathrm{tr}(XX),\ \mathrm{tr}(ZX)\mathrm{tr}(ZX)$

$$\mathcal{D}_2\mathrm{tr}(ZZ)\mathrm{tr}(XX) = \lambda\left(\frac{16}{N}\mathrm{tr}(ZZXX) - \frac{16}{N}\mathrm{tr}(ZXZX)\right)$$

$$\mathcal{D}_2 = \lambda\begin{pmatrix} 4 & -4 & 0 & 0 \\ -8 & 8 & 0 & 0 \\ \frac{16}{N} & -\frac{16}{N} & 0 & 0 \\ -\frac{8}{N} & \frac{8}{N} & 0 & 0 \end{pmatrix}$$

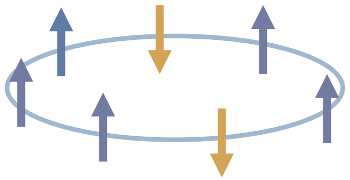
1.1 Operator mixing in $\mathcal{N} = 4$ SYM theory

$$\mathcal{D}_2 = -\frac{2\lambda}{N} : \mathrm{Tr}([X, Z][\check{X}, \check{Z}]) :$$

planar limit

$\mathcal{D}_2$ maps to integrable XXX Hamiltonian $\rightarrow$ diagonalisation via Bethe ansatz techniques [Minahan, Zarembo '02].

$\mathrm{tr}(ZZZXZZX)$

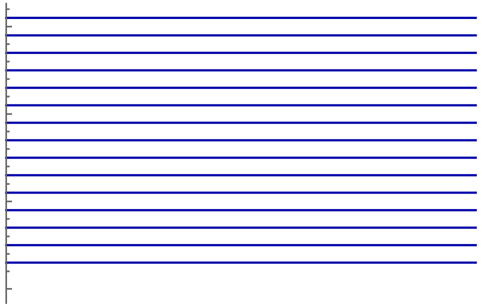


finite N

Do not know how to analytically diagonalise in general case. Here: analyse *numerical* spectrum obtained from direct diagonalisation.

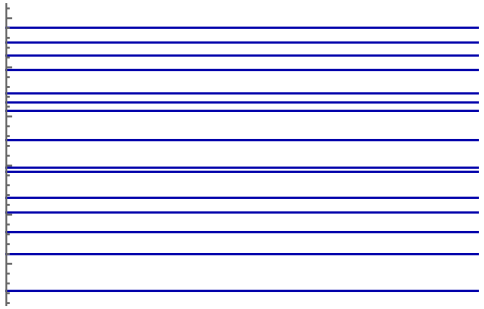
Here: analyse planar & non-planar spectra of SYM theories

1.2 Integrability and chaos from the spectrum



uniform spectrum

e.g. harmonic oscillator



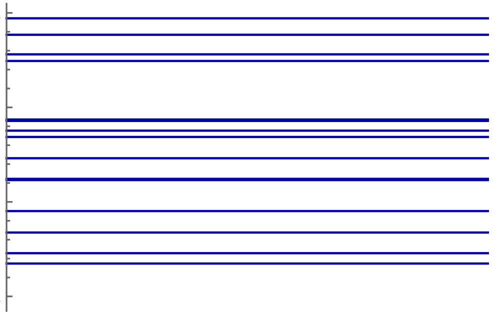
RMT (GOE)

e.g. heavy nuclei

[Wigner ‘51]

[Bohigas, Giannoni, Schmidt ‘84]

quantum chaos



uncorrelated spectrum

e.g. XXX spin chain

[Poilblanc, Ziman, Bellissard, Mila, Montabaux ‘93]

[Berry, Tabor ‘77]

integrability

Correlations in spectra give insight into the nature of the underlying model, in particular the existence/absence of integrability and chaos.

Overview

2.1 Desymmetrisation & Leigh-Strassler spin chain

The (planar) $\mathcal{N} = 4$ SYM theory spectrum is full of degeneracies!

e.g. in the $\mathfrak{su}(2)$ sector:

- residual $SU(2)$ R-symmetry
- parity
- number of traces

Desymmetrisation: look at sectors of fixed global quantum numbers to avoid symmetry-induced degeneracies

2.1 Desymmetrisation & Leigh-Strassler spin chain

For better statistics and as an interesting generalisation study planar Leigh-Strassler theories [Leigh, Strassler '95]

- $\mathcal{N}=1$ supersymmetric exactly marginal deformations of $\mathcal{N}=4$ SYM theory

$$W=\frac{2\lambda'}{1+q}\mathrm{Tr}\left(\Phi_0\Phi_1\Phi_2-q\Phi_1\Phi_0\Phi_2+\frac{h}{3}(\Phi_0^3+\Phi_1^3+\Phi_2^3)\right)$$

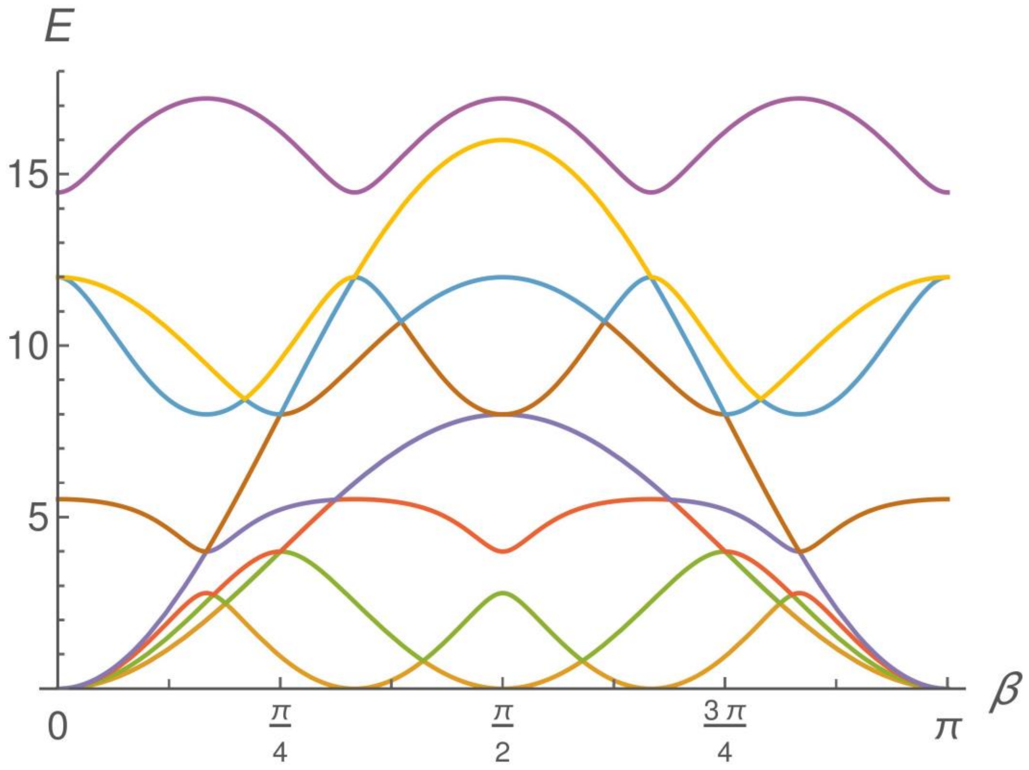
- planar Hamiltonian in chiral sector $\{\phi_1,\phi_2,\phi_3\}$: $H=\sum_l H^{l,l+1}$ $\quad E_{ij}\,|k\rangle=\delta_{jk}\,|i\rangle$

$$\begin{aligned}H^{l,l+1}=&E^l_{i,i}E^{l+1}_{i+1,i+1}-qE^l_{i+1,i}E^{l+1}_{i,i+1}-q^*E^l_{i,i+1}E^{l+1}_{i+1,i}+qq^*E^l_{i+1,i+1}E^{l+1}_{i,i}\\&-qh^*E^l_{i+1,i+2}E^{l+1}_{i,i+2}-q^*hE^l_{i+2,i+1}E^{l+1}_{i+2,i}\\&+hE^l_{i+2,i}E^{l+1}_{i+2,i+1}+h^*E^l_{i,i+2}E^{l+1}_{i+1,i+2}+hh^*E^l_{i,i}E^{l+1}_{i,i}\end{aligned}$$

[Bundzik, Månsson '05]

- Integrable points: $(q,h)=\{(e^{i\beta},0),(0,1/\bar{h}),((1+\rho)e^{2\pi im/3},\rho e^{2\pi in/3}),(-e^{2\pi im/3},e^{2\pi in/3})\}$

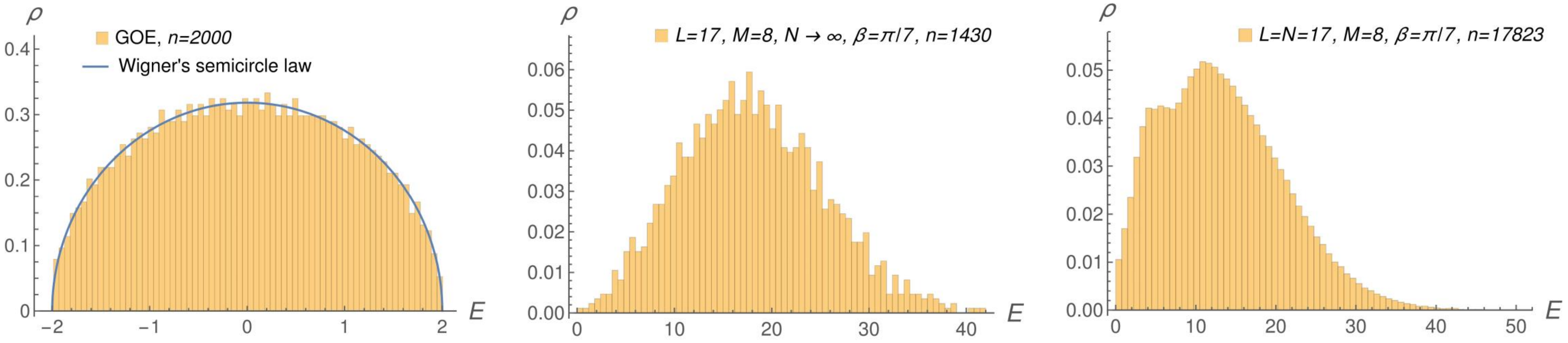
2.1 Desymmetrisation & Leigh-Strassler spin chain



planar eigenvalues of operators in the $\mathfrak{su}(2)$ sector with $L = 6,\ M = 3$ in the β-deformed theory

larger sectors due to reduced symmetry

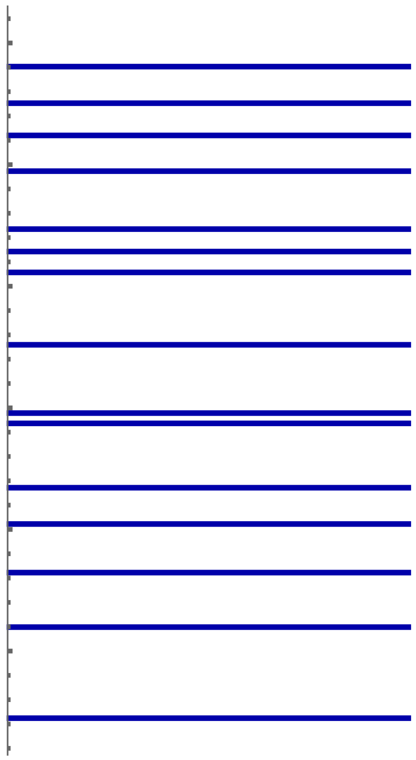
2.2 Unfolding

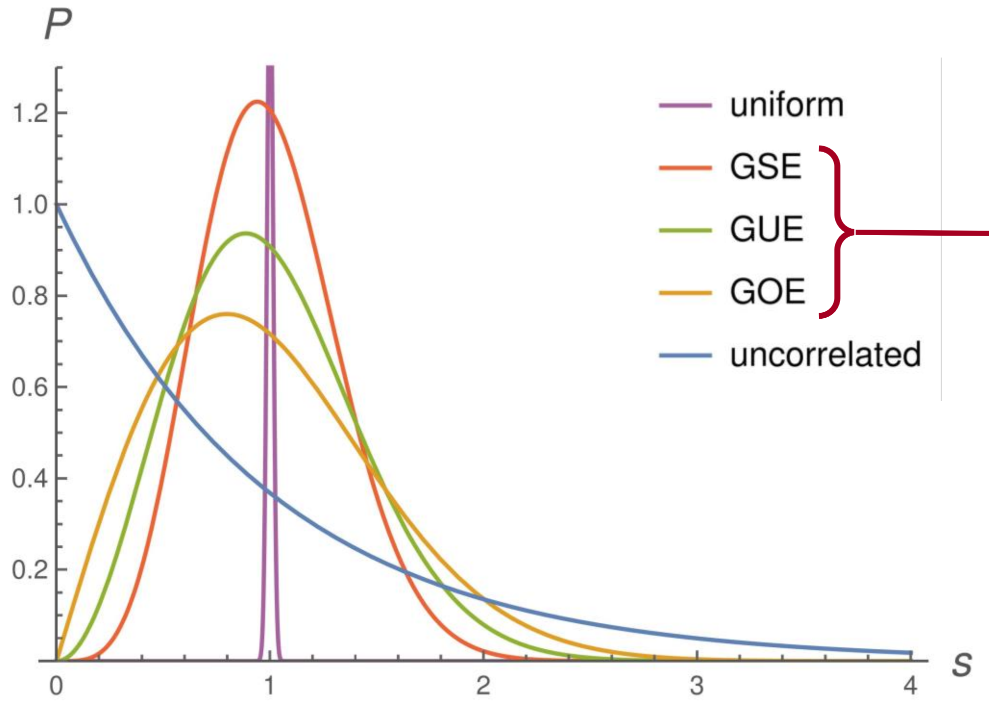


Unfolding: removes the average trend & resulting unfolded spectrum captures the physics of spectral fluctuations

2.2 Level-spacing distributions

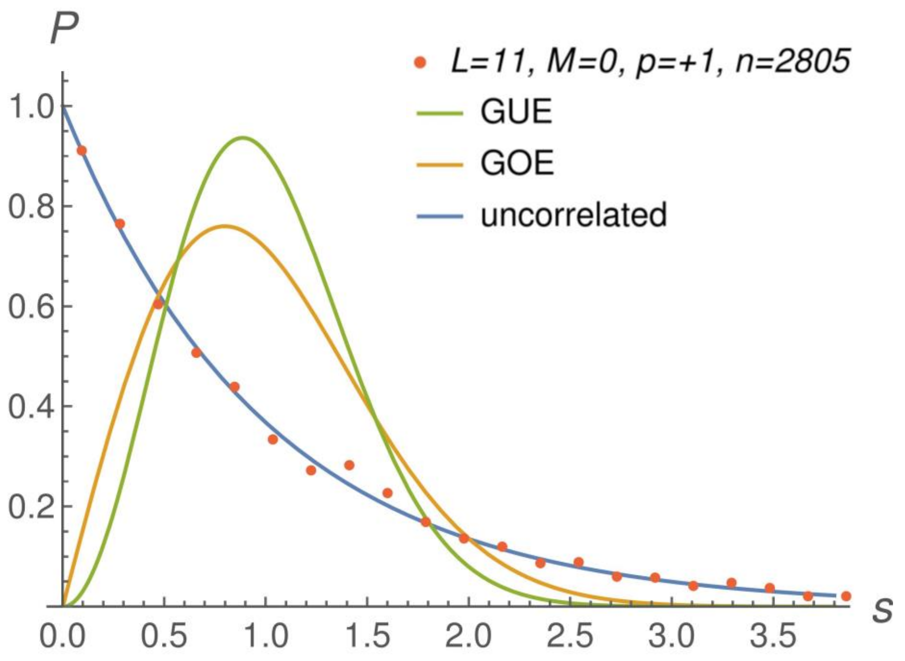
Measure for level correlations: nearest-neighbour spacing distribution



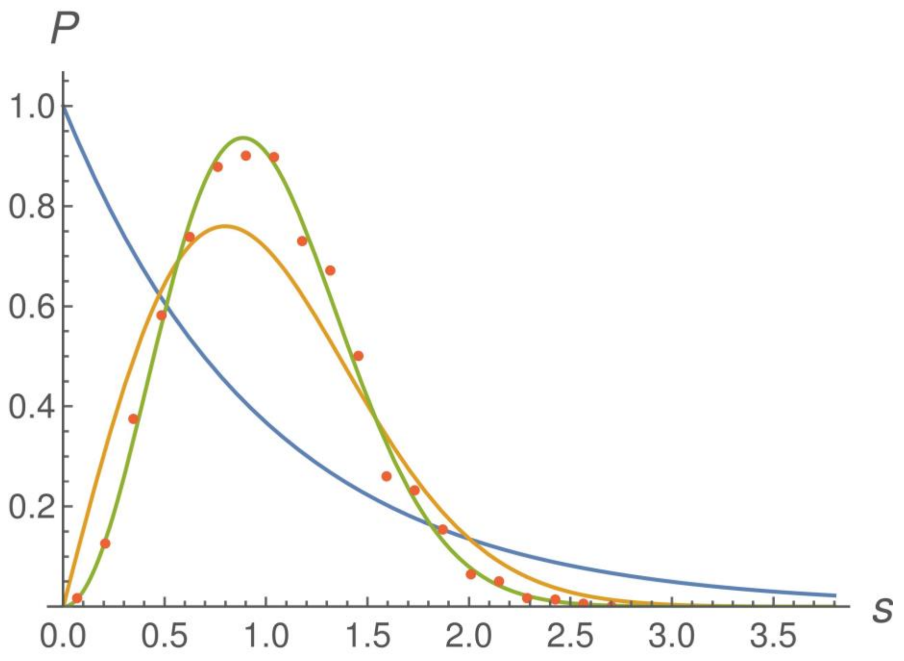


Gaussian
symplectic
unitary
orthogonal
ensemble

2.3 Results from level-spacing analysis



$(q = 0, h = e^{2i/5})$



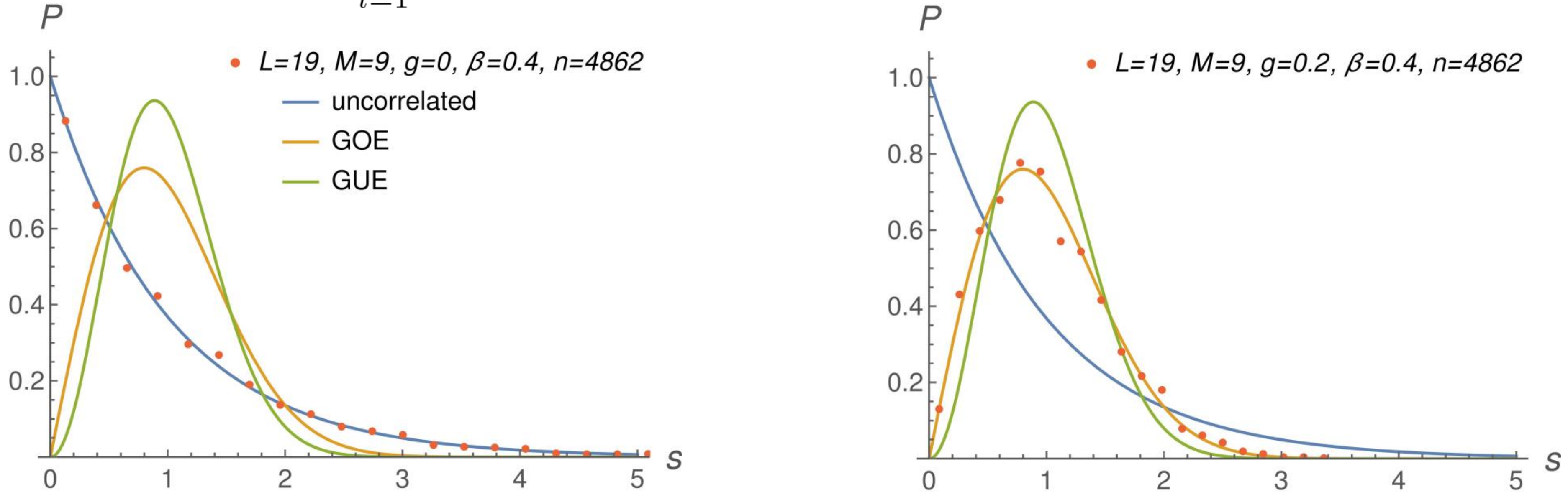
$(q = e^{11i/3}, h = \frac{1}{3}e^{(5+7i)/10})$

at integrable points: Poisson
at non-integrable points: GUE

2.3 Results from level-spacing analysis

Two-loop $\mathcal{N} = 4$ spin chain in $\mathfrak{su}(2)$ sector

$$H = \sum_{l=1}^{L} \left((1+g)\mathbb{1}^{l,l+1} - 4(1-4g) S^l \otimes S^{l+1} - 4g S^l \otimes S^{l+2} \right)$$



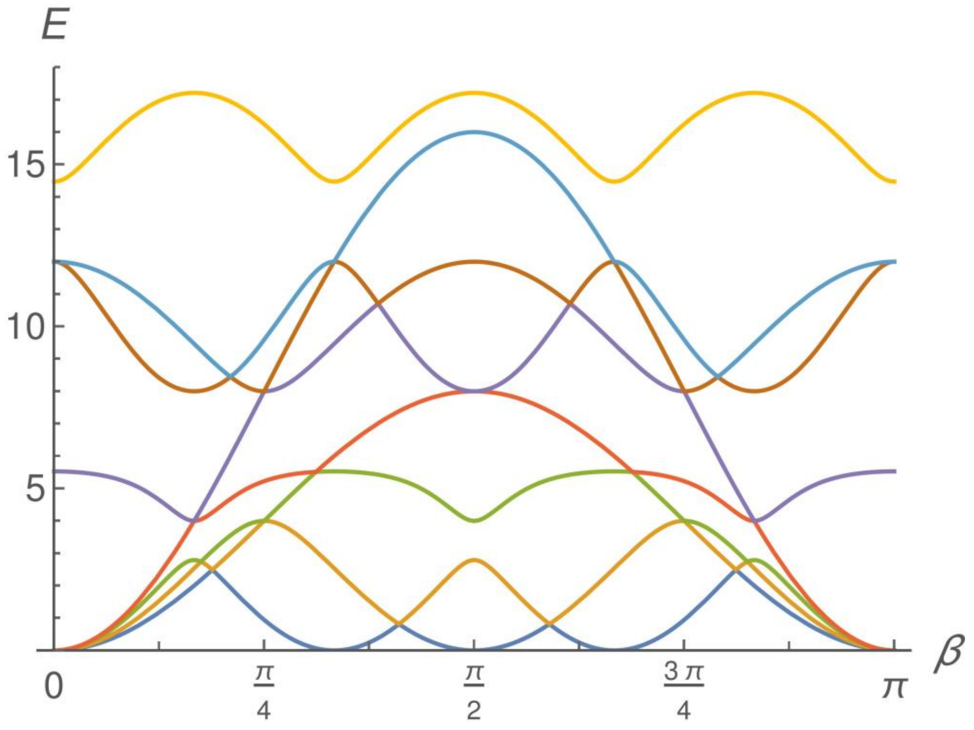
Which specific RMT ensemble appears is related to the presence of discrete symmetries:

- GOE: time-reversal & rotational invariance, or more generally $T = CK$ with $KK^* = 1$
- GUE: no time-reversal invariance

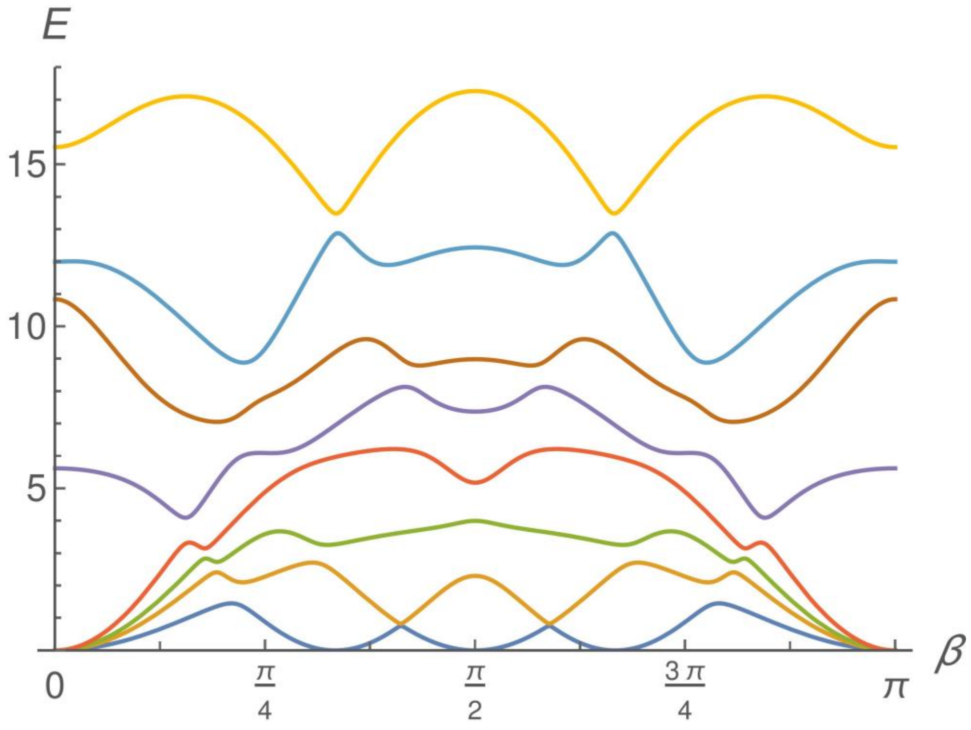
Overview

3.1 Level statistics from the spectrum

$$\mathcal{D}_2 = -\frac{2\lambda_\beta}{N}\left(: \mathrm{Tr}([X,Z]_\beta[\check{X},\check{Z}]_\beta) : -\frac{1}{N} : \mathrm{Tr}[X,Z]_\beta \mathrm{Tr}[\check{X},\check{Z}]_\beta :\right)$$



$L = 6, M = 3, N \to \infty$

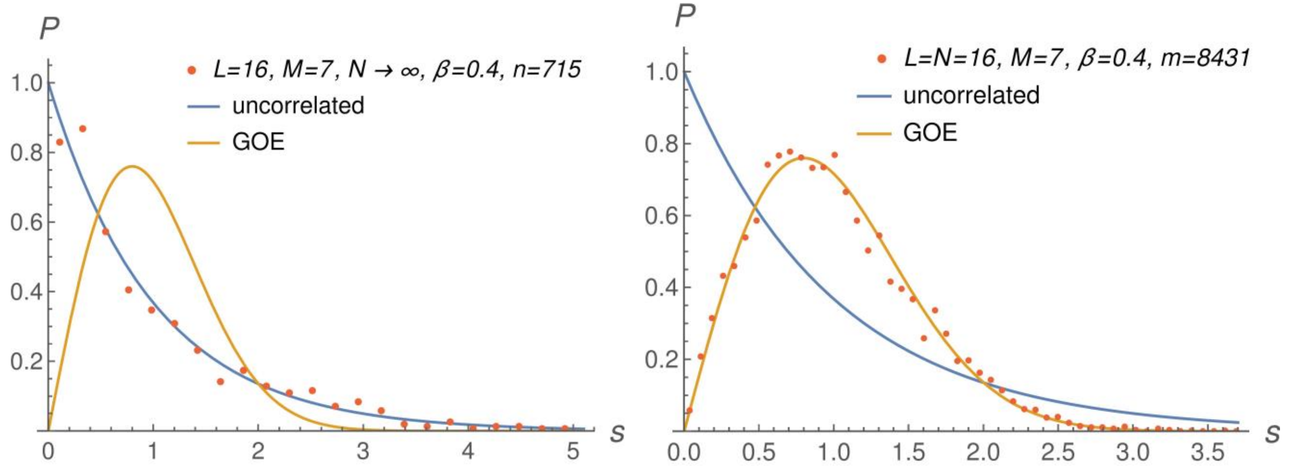


$L = 6, M = 3, N = 7$

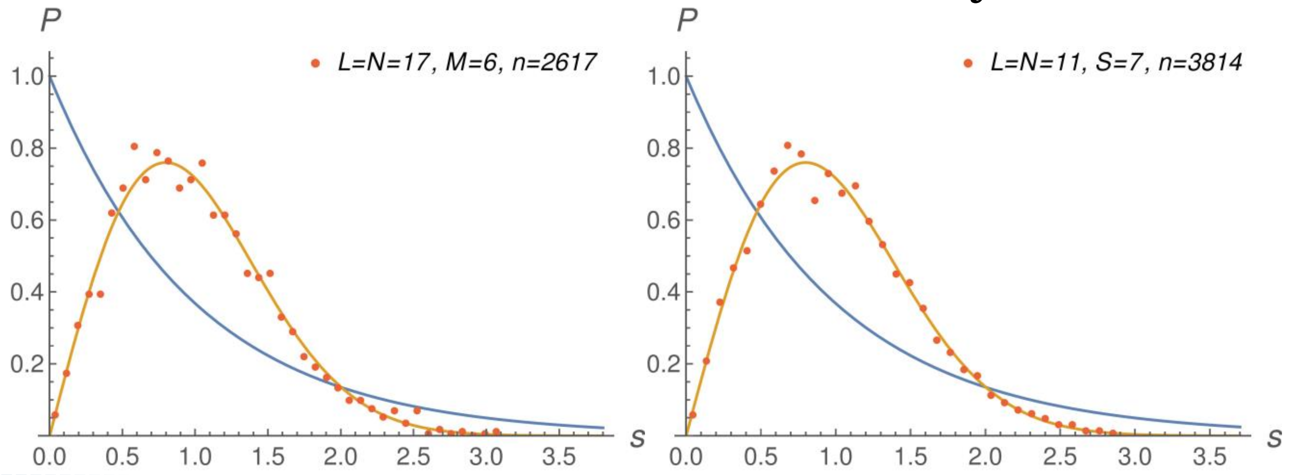
when going away from the planar limit: level repulsion

3.1 Level statistics from the spectrum

- Results for β-deformed $\mathcal{N} = 4$ SYM theory



- Results for undeformed $\mathcal{N} = 4$ SYM theory



planar spectrum
~ Poisson distribution
↓
integrability

non-planar spectrum
~ RMT (GOE)
↓
quantum chaos

3.2 Level statistics from the eigenvectors

Information entropy with respect to a reference basis $|E_k\rangle = \sum_{a=1}^{n} c_{ka} |a\rangle$

$$S_k = -\sum_{a=1}^{n} |c_{ka}|^2 \ln |c_{ka}|^2$$

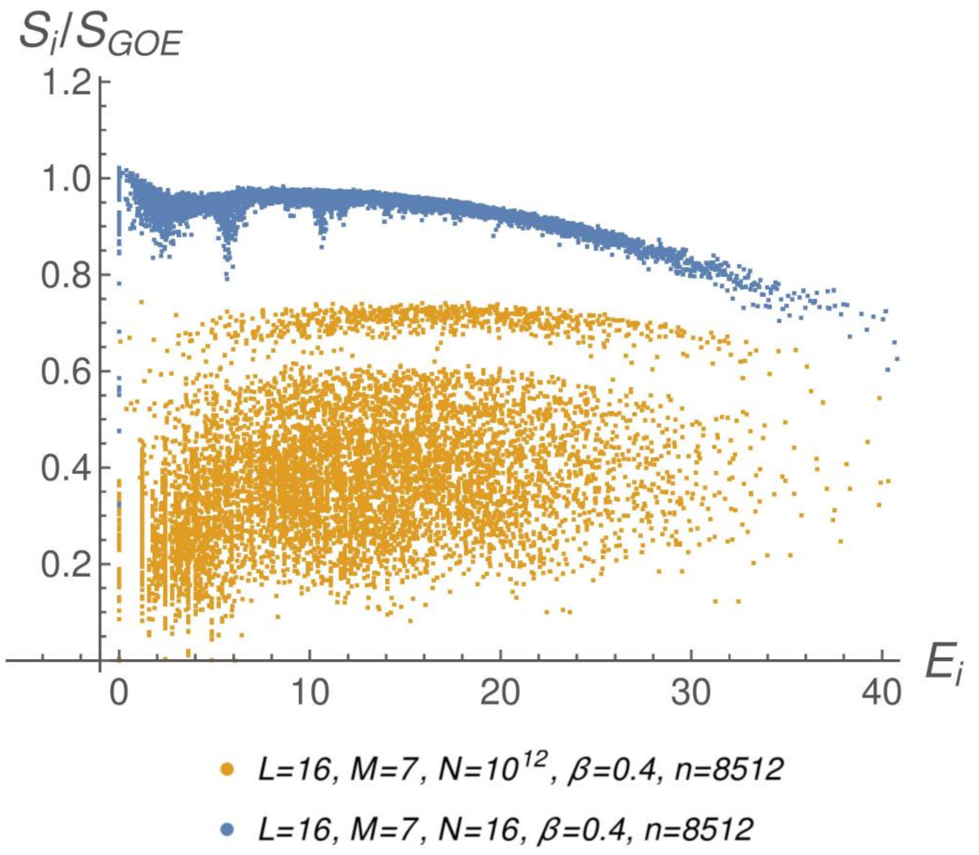
- evenly spread: $|E_k\rangle = \frac{1}{\sqrt{n}} \sum_{a=1}^{n} |a\rangle$

$$S_k = \ln(n)$$

- exponentially localised over $1 \ll m \ll n$ basis states:

$$S_k = \ln(e \cdot m) + \mathcal{O}(m^{-1})$$

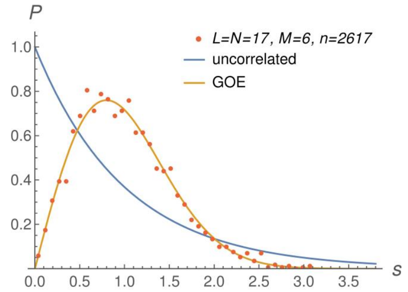
- GOE: $S_k^{GOE} = \ln(2e^{\gamma_E - 2} n) + \mathcal{O}(1/n)$

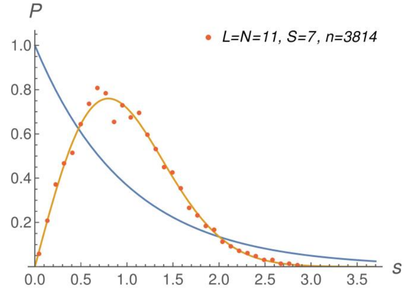


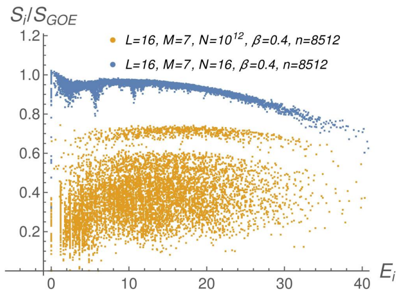
Overview

4. Conclusions & Outlook

- We can test spin chains for integrability using a level statistics analysis, in particular see the existence/absence of integrability for Leigh-Strassler spin chains.
- Finite N spectra in (β-deformed) $\mathcal{N}=4$ SYM theory can be described by RMT







Conjecture:
$\mathcal{N}=4$ SYM theory and its β-deformed version are chaotic at finite N and described by GOE RMT

- Similar behaviour at small N
- Statistics of the numerical spectrum gives insight into the underlying CFT

4. Conclusions & Outlook

- Extend results to other sectors, gauge groups, theories
- More fine-grained observables?
- Basis of (restricted) Schur polynomials more natural at finite N
  [Corley et al '02] [Bhattacharyya et al '08]
- Connect with properties of the dual gravity theory?
  e.g. late-time fluctuations of large AdS black holes in SYK model described by RMT
  [Cotler et al '16]