Exact result on irrelevant deformations of QFTs

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Holographic baby universes

MONICA JINWOO KANG

CALTECH/KAIST

arXiv: 2005.07189, 2006.14620 (MJK, Gesteau)

Baby Universes (a la Giddings-Strominger)

- baby universe $\longrightarrow$ physics of spacetime wormholes
- creation & annhilation of baby universes $\xrightarrow{\text{contribute}}$ Hilbert spaces
- third-quantized model : topology changes in the bulk (spacetime)
- an axion coupled to gravity $\rightarrow$ topology changes $\rightarrow$ baby universes

Lorentzian vs Euclidean Baby Universes

$\Rightarrow$ diffeomorphism invariant

Hamiltonian description:

4d Lorentzian $\Rightarrow$ 3d Euclidean geometries in time direction.

Wheeler-de Witt: $\underbrace{H|\Psi\rangle}_{} = 0$

$\uparrow$

Hartle-Hawking state

$\rightarrow$ superposition of 3d geometries $\Rightarrow$ ensemble!

Algebraic approach to QFT / QG

Bulk / Boundary operators in AdS / CFT

smeared over the entire spatial slice
or a compact spatial subregion

defined only on a code subspace of
the physical $\mathcal{H}$ of the CFT.

consider a general $\mathcal{H}$

→ finite or infinite dimensional

operator algebra

↓ Quantum Error Correction perspective

QFT

Von Neumann algebra

local operator algebras of QFTs

= (infinite-dim) von Neumann algebra $M \subset B(\mathcal{H})$

the set of bounded operators

↓

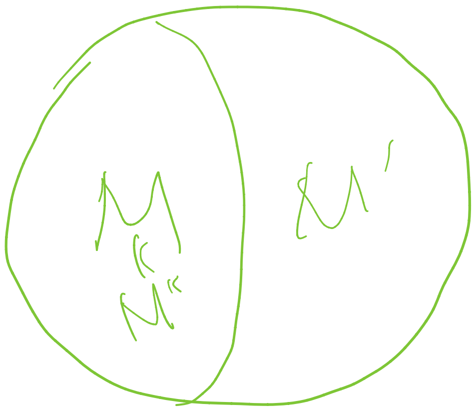
an algebra of bounded operators that

1) contains the identity

2) closed under Hermitian conjugation (∗-algebra)

3) $M = M''$

Commutant $M' = \{ O \in B(\mathcal{H}) \mid OP = PO \ \forall P \in M \}$



Von Neumann algebra & state-dependence

local operator algebras of QFTs

= (infinite-dim) von Neumann algebra

With a cyclic & separating state (or a ground state $|\Omega\rangle \in \mathcal{H}$)

Von Neumann algebra $\longrightarrow$ operator algebra on $\mathcal{H}$

State-dependent !

State-Pushing vs Operator-Pushing

State-Pushing

$\downarrow$

bulk states are mapped to boundary states

Operator-Pushing

$\downarrow$

operators acting on the bulk algebra are directly mapped to boundary operators

ex) Tensor Networks

Before: vN algebra of operators acting on $\mathcal{H}$.

(with states)

Before: vN algebra of operators acting on $\mathcal{H}$.
(with states)

a complex algebra with a norm & an involution

Now: C^*-algebra w/o a notion of $\mathcal{H}$

for operator algebra, use operator-pushing map,
and then construct $\mathcal{H}$ via GNS representation.

Dictionary between operator algebras & quantum gravity

Operator algebras	Quantum gravity
bulk C^*-algebra ($\mathcal{A}_{code}$)	local operators on the bulk
boundary C^*-algebra ($\mathcal{A}_{phys}$)	local operators on the boundary
C^* isometry (ι)	operator pushing map
a strongly-continuous 1-parameter group of isometries of $\mathcal{A}_{phys}$ (σ_t)	Hamiltonian evolution
a KMS state (ω)	a thermal vacuum
commutant of the bulk vN algebra ($\mathcal{M}'_{code}$)	the other side of the wormhole
commutant of the boundary vN algebra ($\mathcal{M}'_{phys}$)	the other boundary

Dictionary between operator algebras & quantum gravity - EX

Operator Algebras	Tensor Networks (cf. HaPPY code)
bulk C^*-algebra ($\mathcal{A}_{code}$)	local operators on the bulk nodes
boundary C^*-algebra ($\mathcal{A}_{phys}$)	local operators on the boundary nodes
C^* isometry (ι)	operator pushing map
a strongly-continuous 1-parameter group of isometries of $\mathcal{A}_{phys}$ (σ_t)	T N Hamiltonian evolution
a KMS state (ω)	a thermal vacuum
commutant of the bulk vN algebra ($\mathcal{M}'_{code}$)	the other side of the wormhole
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Construction with C*- algebra

$$
\begin{tikzcd}[column sep=huge, row sep=large]
\boxed{\begin{array}{c}\text{C*-algebra on the bulk}\\ \text{+ the pull-back of a KMS state}\end{array}} \arrow[r, "\text{GNS representation}", "\text{Construct } \mathscr{H}_{code}"'] \arrow[d, "\iota\ (\text{C* isometry})"] & \boxed{\begin{array}{c}\text{C*-algebra with } \mathscr{H}_{code}\\ \text{+ a thermal state}\end{array}} \arrow[d, "u"] \\
\boxed{\begin{array}{c}\text{C*-algebra on the boundary}\\ \text{+ a KMS state}\end{array}} \arrow[r, "\text{GNS representation}", "\text{Construct } \mathscr{H}_{phys}"'] & \boxed{\begin{array}{c}\text{C*-algebra with } \mathscr{H}_{phys}\\ \text{+ a thermal state}\end{array}}
\end{tikzcd}
$$

Guaranteed by GNS due to the pull-back of the KMS state

GNS construction

Prepare a state : $\mathcal{A}$: C^*- algebra. Say $A \in \mathcal{A}$

a state on $\mathcal{A}$ = a linear functional ω on $\mathcal{A}$ s.t.

$$\begin{cases} \omega(A^\dagger A) \text{ is a nonnegative real number} \\ \omega(\mathrm{Id}) = 1 \end{cases}$$

Want to build $\mathcal{H}$.

Most intuitive idea : Define the inner product with ω

& get the Hilbert space from it.

Most intuitive inner product:

$$\langle B, A \rangle := \omega(B^\dagger A) \quad \text{for } A, B \in \mathcal{A}$$

problem! There can exist observables for which $\omega(A^\dagger A) = 0$.

$\rightarrow$ quotient these out

$$I := \{ A \in \mathcal{A},\ \omega(A^\dagger A) = 0 \}$$

$\rightarrow$ Then we get $\mathcal{H}_\omega := \mathcal{A} / I$,

where $\mathcal{A}$ acts by left multiplication.

$\rightarrow$ GNS representation of $\mathcal{A}$.

Key point:

The GNS representation π_ω of a state ω is that it purifies ω into the vector state $|[Id]\rangle$.

$$\boxed{\omega(A) = \langle [Id] \,|\, \pi_\omega(A) \,|\, [Id] \rangle} \quad \forall A \in \mathcal{A}.$$

Consider a particular set of states : KMS state

characterizes thermal equilibrium w.r.t. a time evolution

→ Recall for a finite-dimensional system : n-dim

- algebra ($\mathcal{A}_n$) of n-dim matrices.
- a Hamiltonian (H) generating the time evolution.

$$\sigma_t(A) := e^{iHt} A e^{-iHt}$$

- the Gibbs state $\rho_\beta := \dfrac{e^{-\beta H}}{\mathrm{Tr}(e^{-\beta H})}$ is the unique state representing thermal equilibrium at β.

Generalize to include for infinite-dim

→ traces are not always well-defined.

→ can no longer compute the Gibbs state.

→ Instead, we have KMS condition.

- C^*-algebra : $\mathcal{A}$
- a state on $\mathcal{A}$: ω
- a 1-parameter group of automorphisms of $\mathcal{A}$: σ_t

→ For $\beta > 0$, ω is a KMS_β state if $\exists\, F_{AB}$ analytic on the strip $\{0 < \operatorname{Im} z < \beta\}$ & continuous on its closure s.t. $F_{AB}(t) = \omega(A\sigma_t(B))$ & $F_{AB}(t+i\beta) = \omega(\sigma_t(A)B)$.

Key point:

The GNS representation allows to represent a KMS (thermal) state as a vector state on a Hilbert space.

Consider TFD construction.

- finite-dim $\mathcal{H}$
- thermal density matrix $\rho_\beta := \dfrac{e^{-\beta H}}{\mathrm{Tr}\, e^{-\beta H}}$
- TFD doubles the size of the Hilbert space to construct a purification of the mixed ρ_β.

$$\Rightarrow \mathcal{H} \otimes \mathcal{H} \quad (\text{doubled Hilbert space})$$

- In $\mathcal{H}\otimes\mathcal{H}$, the vector state is constructed as

$$|TFD\rangle_\beta := \sum_i e^{-\beta E_i/2} |e_i\rangle \otimes |e_i\rangle$$

eigenbasis of the Hamiltonian

defines a purification of ρ_β.

Consider a finite-dim context with $\mathcal{H}_n$ (n-dim Hilbert space)

WLOG, assume the thermal density matrix is nondegenerate.

- For a Gibbs state, the thermal density matrix is invertible.
  → the ideal of the GNS representation is trivial.
- The Hilbert space is directly constructed out of the algebra $M_n(\mathbb{C})$, which is isomorphic to $\mathcal{H}_n \otimes \mathcal{H}_n$
- The density matrix ρ_β is now represented as a vector state.

⇒ The GNS representation is exactly the same as the TFD construction in the nondegenerate finite-dimension case.

key point:

TFD construction transforms any mixed thermal state into a vector state

$\Rightarrow$ This is exactly what GNS representation of a KMS state does.

So far we established :

1. Hilbert spaces of bulk & boundary
2. a bulk-to-boundary (Hilbert space) isometry
3. a vector state for each Hibert space
   (TFD state generalization).

we still need to formulate : von Neumann algebra

→ local operator algebra of QFT

- The GNS representation π_ω purifies a state ω into the vector state $|[\mathrm{Id}]\rangle$

$$\boxed{\omega(A) = \langle [\mathrm{Id}] | \pi_\omega(A) | [\mathrm{Id}] \rangle}$$

→ cyclic

- The action of $\mathcal{A}$ on the state spans a norm-dense subset of $\mathcal{H}$.

→ cyclic property extends to the bi-commutant of the C^*-algebra on the GNS Hilbert space.

⇒ Generates a vN algebra!

- The GNS representation π_ω purifies a state ω into the vector state $|[\mathrm{Id}]\rangle$

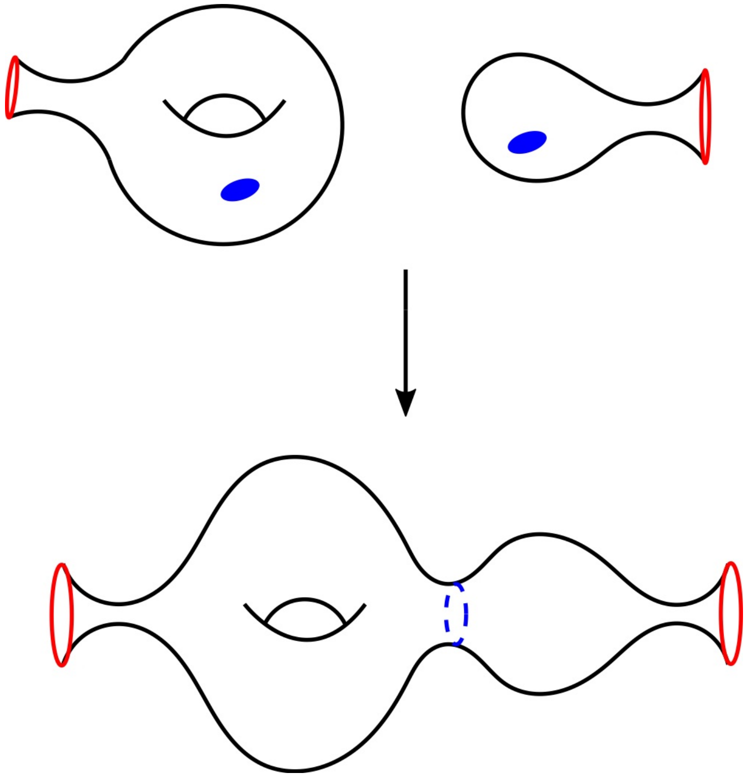
$$\boxed{\omega(A) = \langle [\mathrm{Id}] | \pi_\omega(A) | \underline{[\mathrm{Id}]} \rangle} \to \text{cyclic}$$

- The action of $\mathcal{A}$ on the state spans a norm-dense subset of $\mathcal{H}$.
  - → cyclic property extends to the bi-commutant of the C*-algebra on the GNS Hilbert space.
  - ⇒ Generates a vN algebra with a cyclic & separating state!
- The vector representative $|\Psi\rangle$ of a KMS state
  - ⇒ a separating vector for the C*-algebra $\mathcal{A}$
  - ⇒ this property holds for the whole vN algebra closure $\mathcal{A}''$

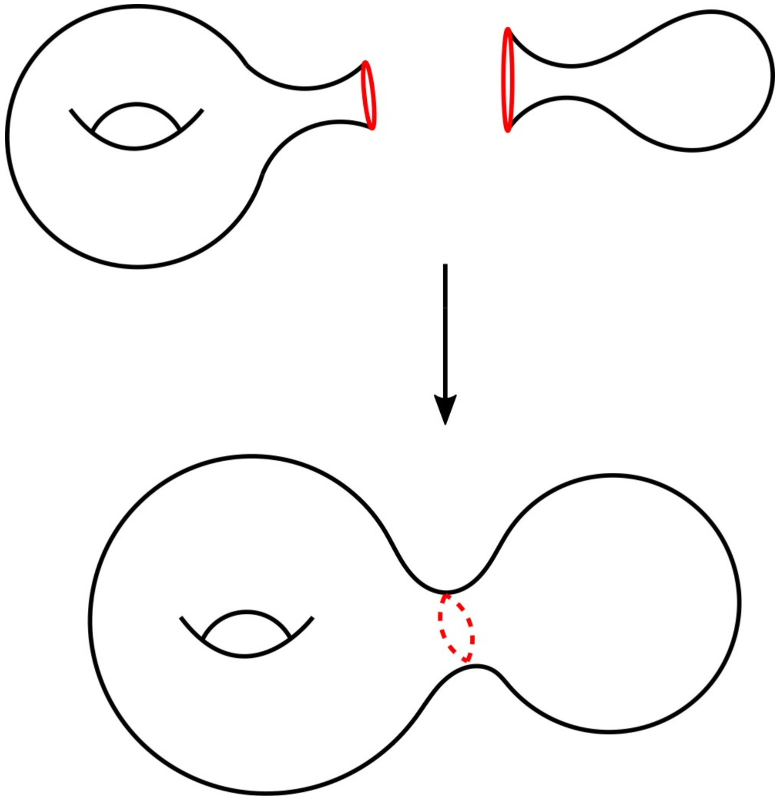
Apply the GNS construction to

the baby universes.

Baby Universe Operations

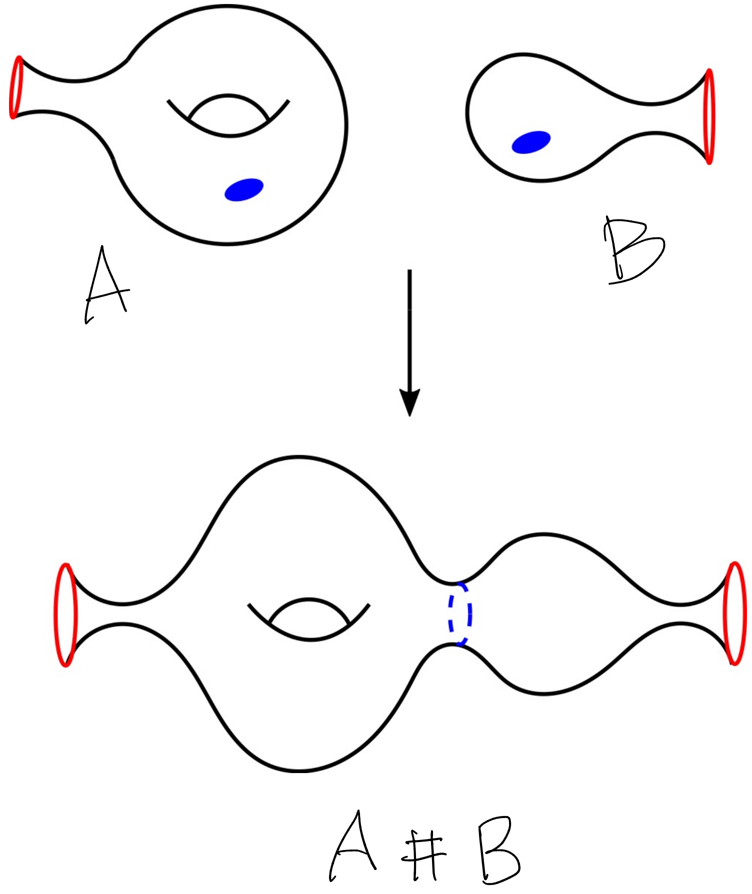


connected sum



gluing

Baby Universe Operations



connected sum $A \# B$

→ baby universe (U) creation

→ two scenarios

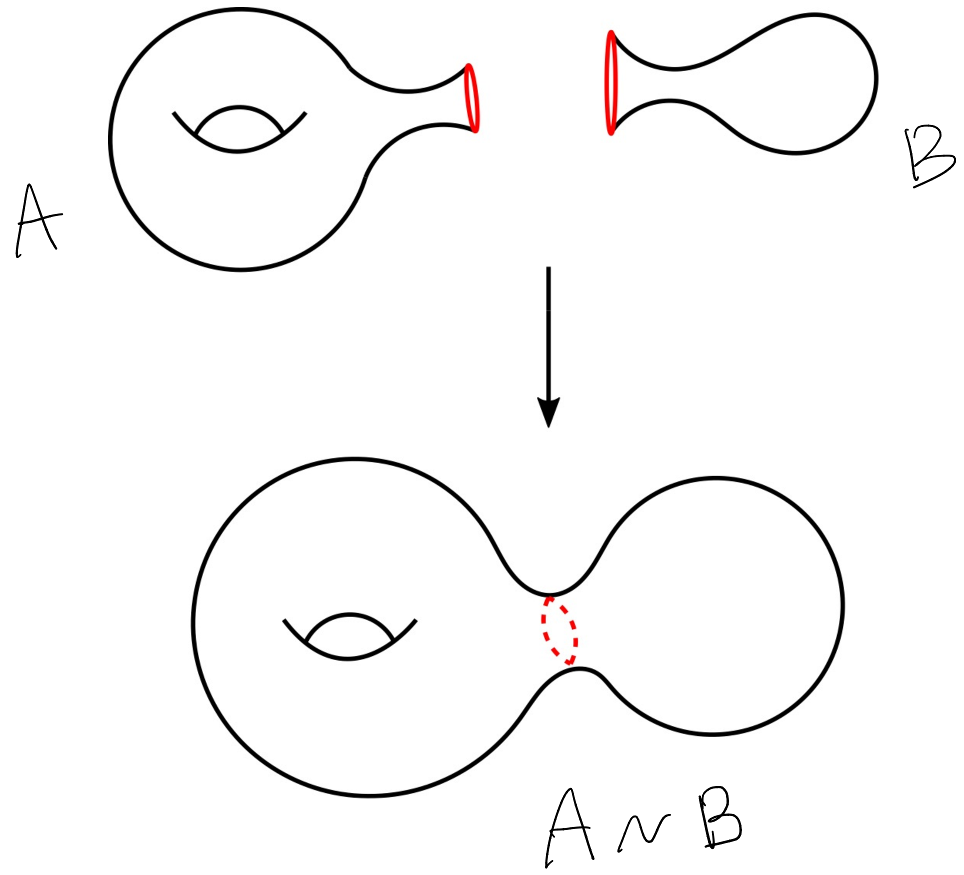
1) connected sum with a pre-existing piece (P) of a spacetime manifold

→ $U \# P$

2) lives on its own

→ $U \# \phi$

Baby Universe Operations



gluing operation A ∼ B

→ baby universe annhilation

Baby Universe Operations & Diffeomorphism Invariance

↳ act at the level of topology of the space-time manifold

→ many ways to perform them? Not quite!

→ these operations inherit a nice structure due to diffeomorphism invariance

ex) the connected sum of two manifolds can be uniquely defined in a diffeomorphism invariant theory.

⇒ large number of possibilities on performing a connected sum is completely gauged out by diffeomorphism invariance.

→ these operations are mutually associative & commutative upto diffeomorphism.

⇒ the outcome of these operations performed in a row will NOT depend on the order or the place where they are performed.

Baby Universe Algebra

- The algebra of observables of bulk quantum gravity

$$\mathcal{A}_{QG} := \underbrace{\mathcal{A}_{res}}_{\text{reservoir algebra}} \otimes \underbrace{\mathcal{A}_{baby}}_{\text{Abelian baby universe algebra}}$$

↳ Abelian baby universe algebra
(∵ the properties of the baby universe operations)

reservoir algebra $\left(\mathcal{A}_{res} := \bigotimes_{b} \bigotimes_{n=1}^{\infty} \mathcal{A}_b^n \otimes \bigotimes_{b} \bigotimes_{n=1}^{\infty} \overline{\mathcal{A}_b^n}\right)$

b → different boundary structures

$\overline{\mathcal{A}_b^n}$ → conjugate of $\mathcal{A}_b^n$

A reference state to the Hilbert space with a vacuum

- Want to apply the GNS construction
  → require a reference state (KMS state)
- perform GNS representation w.r.t. an abstract state on the algebra of observables
  = expectation value functional
  → gravitational path integral
  "summing over geometries"
- Apply GNS construction : $\exists$ a vacuum $\langle\Omega|A|\Omega\rangle = \omega(A)$,
  the Hilbert space : $\mathcal{H}_{baby} = \mathcal{A}_{baby} / I$ with $I := \{A \in \mathcal{A}_{baby} , \ \omega(A^* A) = 0\}$

Recall: $\mathcal{A}_{baby}$ is a commutative C^*-algebra.

ω is a state on $\mathcal{A}$.

Under this setting,

$\boxed{\text{The GNS representation of } \mathcal{A}_{baby} \text{ (arising from } \omega) \text{ has dimension one}}$

$\Updownarrow$ (Størmer)

$\boxed{\text{The GNS representation of } \mathcal{A}_{baby} \text{ (arising from } \omega) \text{ is irreducible}}$

What does it mean for a state on a C^*-algebra to have an irreducible representation?

the space of states on a C^*-algebra

= the space of expectation value functionals on this algebra

a convex space $\Rightarrow$ has extremal points

i.e. points that cannot be expressed as a convex combination of two other ones

= pure state

(cannot be expressed as a statistical mixture of other states)

More concretely $\Rightarrow$

Recall : $\mathcal{A}_{baby}$ is a commutative C^*- algebra.

ω is a state on $\mathcal{A}$.

Under this setting,

The GNS representation of $\mathcal{A}_{baby}$ (arising from ω) has dimension one

$\Updownarrow$ (Stormer)

The GNS representation of $\mathcal{A}_{baby}$ (arising from ω) is irreducible

$\Updownarrow$

ω is a pure state

all equivalent !

More precisely:

the space of states on a C^*-algebra is a bit more than a convex space

Further has a property:

every ω on $\mathcal{A}_{\text{baby}}$ can be written as $\omega = \int_{\varphi \text{ pure}} \varphi \, d\mu(\varphi)$.

More precisely:

the space of states on a C*-algebra is a bit more than a convex space

Further has a property:

every ω on $\mathcal{A}_{\text{baby}}$ can be written as $\omega = \int_{\varphi \text{ pure}} \varphi \, d\mu(\varphi)$

In terms of GNS representation: $\mathcal{H}_\omega = \int^{\oplus} \mathcal{H}_\varphi d\mu(\varphi)$

irreducible GNS representations

(for here, dim-1 Hilbert spaces)

$\Rightarrow$ corresponds to a member of the ensemble of theories averaged over

More precisely:

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irreducible GNS representations

(for here, dim-1 Hilbert spaces)

$\Rightarrow$ corresponds to a member of the ensemble of theories averaged over

this ensemble contains one element

$\Updownarrow$

ω is pure

The GNS representation of $\mathcal{A}_{baby}$ has dimension one if and only if it is NOT an ensemble!

Finding the minimal restriction to dimension-one

Assume the restriction of this state (ω) to $\mathcal{A}_{baby}$ is pure.

$\Rightarrow \quad \omega = \omega_{res} \otimes \omega_{baby}$

What does this mean for the gravitational path integral?

$\Rightarrow$ It factorizes: $Z_{QG} = \int_{CFT} D[\phi]e^{-S[\phi]} \int_{baby} D[b]e^{-S[b]}$

$\Rightarrow$ The formation of the baby universes cannot be influenced by the boundary observables.

Put together: Say $A_{QG} = A_{res} \otimes A_{baby}$ (with $\mathcal{A}_{baby}$ abelian).

The GNS representation of $\mathcal{A}_{baby}$ (induced by ω) is dimension one (= the Hilbert space of the baby universe is of dimension one) if and only if the restriction of ω to $\mathcal{A}_{baby}$ is pure.

Put together: Say $\mathcal{A}_{QG} = \mathcal{A}_{res} \otimes \mathcal{A}_{baby}$ (with $\mathcal{A}_{baby}$ abelian).

The GNS representation of $\mathcal{A}_{baby}$ (induced by ω) is dimension one

baby universe hypothesis

(= the Hilbert space of the baby universe is of dimension one)

if and only if the restriction of ω to $\mathcal{A}_{baby}$ is pure.

Or physically speaking,

Assume 1. $\mathcal{A}_{QG} = \mathcal{A}_{res} \otimes \mathcal{A}_{baby}$ for $\mathcal{A}_{baby}$ abelian

2. ω is a pure state on $\mathcal{A}_{QG}$

3. $\omega = \omega_{res} \otimes \omega_{baby}$

Then, the GNS representation of $\mathcal{A}_{baby}$ is one-dimensional.

If such is all satisfied to be true,

what should such a state look like?

Recall again: $\mathcal{A}_{baby}$ is an Abelian C^*-algebra.

ω on $\mathcal{A}_{baby}$ is pure $\Longleftrightarrow$ $\omega(AB) = \omega(A)\omega(B)$ (multiplicative)

→ this state is called a character of $\mathcal{A}_{baby}$.

→ this is equivalent to: the correlation functions of the effects of adding two boundaries factorize.

" miraculous cancellations "

let's see this being granted by character theory

Any Abelian C^*-algebra $\mathcal{A}$

$\simeq$ the algebra of complex valued continuous functions $C_0(X)$
on a locally compact space $\underline{X}$
vanishing @ infinity

X is compact $\Leftrightarrow$ $\mathcal{A}$ contains a unit

$\Rightarrow X =$ Gelfand space of $\mathcal{A}$

The space of pure states (= α-parameters) on $\mathcal{A}$
= evaluation functionals on X $\simeq X$.

The space of characters on $\mathcal{A}$ realize the "miraculous cancellations"
in the gravitational path integral $\Longleftrightarrow$ the Gelfand space of $\mathcal{A}$!

EX : a topological theory (example in Marolf - Maxfield)

one boundary structure ($b=1$)

boundary-inserting operator : $Z^\dagger = Z$

conjugate boundary : indistinguishable

identify Z with a real variable function $f(x)=x$ → invariant under c.c.

a state $\omega = \int_{\mathbb{R}} \varphi d\mu(\varphi)$ ⟶ $\omega = \sum_{n=0}^{\infty} \delta_n P(Z=n)$

GNS ⇒ pre-Hilbert space ⇒ Hilbert space ← collapsing!

Gelfand space $= \mathbb{R}$

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GNS ⇒ pre-Hilbert space ⇒ Hilbert space ← collapsing!

Gelfand space = $\mathbb{R}$ Let's add EOW branes!

EX : a topological theory (example in Marolf - Maxfield)

one boundary structure ($b=1$)

boundary-inserting operator : $Z^\dagger = Z$ hermitian

conjugate boundary : indistinguishable

identify Z with a real variable function $f(x)=x$ → invariant under c.c.

a state $\omega = \int_{\mathbb{R}} \varphi \, d\mu(\varphi) \longrightarrow \omega = \sum_{n=0}^{\infty} \delta_n P(Z=n)$ poisson

GNS ⇒ pre-Hilbert space ⇒ Hilbert space ← collapsing!

Gelfand space (with EOW branes) = the product of $\mathbb{R}$ & a space of matrices

Ex: QCD theory with θ-vacua

boundary-inserting operator : $Z^{\dagger} = Z^{-1}$ <u>unitary</u>

$\Rightarrow$ inherit a group structure!

Call it G = a group of C^*- algebra

Ex: QCD theory with θ-vacua

boundary-inserting operator : $Z^\dagger = Z^{-1}$

$\Rightarrow$ inherit a group structure!

call it G = a group of C^*-algebra

$\Rightarrow$ corresponding group algebra : $\mathbb{C}[G] := \left\{ \sum_{i=1}^{n} \lambda_i g_i , \lambda_i \in \mathbb{C}, g_i \in G \right\}$

consider k boundary structures $(b = k)$

$\Rightarrow G = \mathbb{Z}^k$ (k generators)

$\Rightarrow \mathbb{C}[\mathbb{Z}^k] \simeq$ the algebra of polynomials with k variables & positive/negative powers

endow this with an involution & a norm $\|x^* x\| = \|x\|^2$ $\Rightarrow$ defined $\mathcal{A}_{bdry} = C^*(G)$

taking the group inverse of each group element and the c.c. of each scalar

a group C^*-algebra

Ex: QCD theory with θ-vacua

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Gelfand dual of $C^*(G) = \hat{G}$ (Pontryagin dual)

Ex: QCD theory with θ-vacua

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consider k boundary structures $(b=k)$

$\Rightarrow G = \mathbb{Z}^k$ (k free Abelian generators) $\Rightarrow$ Gelfand dual of $\mathbb{Z}^k \simeq$ a k-torus

$k=1$: a unit circle!

$\Rightarrow \mathbb{C}[\mathbb{Z}^k] \simeq$ the algebra of polynomials with k variables & positive/negative powers

endow this with an involution & a norm $\|x^* x\| = \|x\|^2$ $\Rightarrow$ defined $\mathcal{A}_{bdry} = C^*(G)$

Gelfand dual of $C^*(G) = \hat{G}$ (Pontryagin dual)

Ex: QCD theory with θ-vacua

Consider when $G = \mathbb{Z}^k$

an observable $O \in \mathcal{A}_{BG} = \mathcal{A}_{res} \otimes \mathcal{A}_{baby}$:

$$O = \sum_p a_p \underbrace{\boxed{A_p}}_{\mathcal{A}_{res} \ni} \otimes \underbrace{\boxed{B_p}}_{\mathcal{A}_{baby} \ni}$$

$$\big\downarrow \text{ baby universe action} \qquad g = \sum_{i=1}^{k} n_i g_i$$

k independent generators of G

$$O_g = \sum_p a_p A_p \otimes g B_p$$

a pure state ω = character of $G = \mathbb{Z}^k$

$$\Rightarrow \omega(O_g) = \sum_p \omega(A_p)\omega(g)\omega(B_p) = e^{i\sum_{i=1}^k n_i\alpha_i} \sum_p \omega(A_p)\omega(B_p), \quad \omega(g_i) = e^{i\alpha_i}$$

parametrized by $\alpha_i \in \mathbb{R}$ which span $\hat{G} = U(1)^k$

⇒ the action of the baby universe transformation adds a phase to the action for the gravitational path integral on an observable.

Ex: QCD theory with θ-vacua

$\left\{ \begin{array}{l} \text{the group of baby universe transformations } G \Rightarrow \text{a (global) symmetry group.} \\ \text{associated } \alpha\text{-parameters} \Rightarrow \text{topological charges} \end{array} \right.$

$\longrightarrow$ connect to θ-vacua in non-Abelian gauge theory.

$\pi_3(G) = \mathbb{Z}$ for G any compact connected simple Lie group (cf. QCD)

$\Rightarrow$ phases $e^{in\theta}$

θ: vacuum angle (labels superselection sectors) $\Longleftrightarrow$ α-parameters (gives irreducible GNS representations)

n: winding number $\Longleftrightarrow$ # of boundaries

Outlook & applications for GNS construction-based formalism

- Lorentzian theories
- Tensor networks
- Approximate theorem to EW reconstruction = JLMS
- "Quantum extremal surface" in infinite-dim $\mathcal{H}$
- Construction of bulk utilizing boundaries.

Construct bulk of the Narain Boundary (utilizing Maloney-Witten)

Collection of boundary Riemann surfaces $\Sigma_1, \cdots, \Sigma_n$

of genera $g_1, \cdots, g_n$

each term $\Downarrow$

Lagrangian sublattice of L of $\bigoplus_i H(\Sigma_i)$

$\rightarrow$ construct a compression body w/ $n+1$ boundaries

n of them $\cong \Sigma_i$

$n+1$th one $\cong \Sigma$ of genus $\sum_i g_i$.

Kleinien uniformization

given by the quotient of the Riemann sphere by the free product of n Fuchsian groups, each corresponding to a uniformizing group for one of the Σ_i.

Construct bulk of the Narain Boundary

Note: $\exists$ Lagrangian sublattice L' of Σ s.t. $L'-L$ is contractible in C.

Consider a handlebody S_L w/ boundary s.t. L' is contractible in S.

Gluing S_L & C trivially along Σ gives an explicit topology M_L

w/ boundaries $\Sigma_1, \cdots, \Sigma_n$ for which L is contractible.

$\Rightarrow$ Bulk wormholes via Heegard splitting process.

Now we can perhaps endow geometric data on this M_L via Chern-Simons

line bundle! [Gesteau, MK, Marcolli, work in progress]

Can we consider then a Narain moduli w/ $T\bar{T}$ or $J\bar{T}$ for the CFT & construct bulk?

$\hookrightarrow$ [Chakraborty, Hashimoto]

Thank you for listening!